Towards the quantization of the non-relativistic D2-brane in the Pure Spinor Formalism

A. Herrera–Aguilar[a,b,1], J.E. Paschalis [c,2]

[a]Instituto de Física, Benemérita Universidad Autónoma de Puebla, Apartado Postal J-48, 72570, Puebla, Puebla, México
[b]Instituto de Física y Matemáticas, Universidad Michoacana de San Nicolás de Hidalgo, Edificio C-3,Ciudad Universitaria, CP 58040, Morelia, Michoacán, México
[c] Theoretical Physics Department, Aristotle University of Thessaloniki, 54124 Thessaloniki, Greece

Abstract

The non-relativistic D2-brane is treated in the framework of pure spinor formalism. The fermionic constraints corresponding to the rescaled fermionic coordinates are given. Two commuting spinor fields are introduced, each one corresponding to a fermionic constraint. A BRST charge is constructed via the ansatz proposed by N. Berkovits in [1]–[4]. The nilpotency of the BRST charge leads to a set of constraints for the two spinor fields including pure spinor constraints. A nontrivial solution is given for one of the spinor fields.

[1]Email: aherrera@ifuap.buap.mx
[2]Email: paschalis@physics.auth.gr

1 Introduction

The pure spinor formalism introduced by N. Berkovits [1]–[4] is a successful attempt to solve the longstanding problem of finding a manifestly supersymmetric and covariant superstring formalism. The basic ingredient is the BRST-like operator $Q = \int dz \lambda^\alpha d_\alpha$ where d_α is the fermionic constraint that appears in the conventional Green-Schwarz formalism and λ_α is a bosonic chiral spinor that plays the role of the associated "ghost". For Q to be regarded as a BRST operator must be nilpotent and this leads to the relation $\lambda^\alpha \gamma^m_{\alpha\beta} \lambda^\beta = 0$. This in 10 dimensions is the condition for λ^α to be charactrized as a pure spinor.

An important property of Q is that its cohomology correctly reproduces the spectrum of the superstring. The pure spinor formalism has been used as well for the covariant quantization of the superparticle [3] and also to study several aspects of string theory, for example the propagation of strings in curved backgrounds [5]–[8].

Another important application is the calculation of scattering amplitudes within the framework of superstring theory [9]–[13]. The manifest Lorentz covariance and spacetime supersymmetry make the calculation much easier than in other formalisms. Thus, pure spinors play a crucial role within string perturbation theory. However, within the context of branes (D-branes, superbranes), there are almost no non-trivial solutions reported in the literature. In particular, the Lagrangian formalism for the supermembrane for 11d supergravity backgrounds was constructed in [14] within the pure spinor framework where a solution to the pure spinor constraints was presented as well.

In this paper we try to extend the pure spinor formalism to the case of the nonrelativistic IIA D2-brane. The nonrelativistic limit of string theories [15]–[16] give us a deeper understanding of string theories themselves. The nonrelativistic limit of Dp branes has been studied in [17]–[18]. It is important to note that in this limit the kappa symmetry is maintained and this allows us to treat nonrelativistic Dp branes in the framework of the pure spinor formalism.

Here we present novel non-trivial solution for the non-relativistic D2-brane within the pure spinor formalism. This fact could lead to the quantization of branes with interesting and relevant results.

Our starting point is the action of a IIA D2-brane in a flat 10d background. The fields consist of the 10d superspace coordinates (x^m, θ) and an abelian gauge field A_μ [19]–[21]:

$$S = -T \int d^3\sigma \sqrt{-\det(G_{\mu\nu} + \mathcal{F}_{\mu\nu})} + \int \mathcal{L}_3^{WZ}, \tag{1.1}$$

where T is the string tension and the Wess-Zumino action reads

$$\mathcal{L}_3^{WZ} = T\{\frac{1}{2} d\bar{\theta} \not{\Pi}^2 \theta - \frac{1}{3} d\bar{\theta} \not{\Pi} \theta_1 + \frac{1}{15} d\bar{\theta} \theta_2 + d\bar{\theta} \Gamma_{11} \theta \mathcal{F}\}; \tag{1.2}$$

$G_{\mu\nu} = \eta_{mn}\Pi_\mu^m \Pi_\nu^n$ and here $\Pi_\mu^m = \frac{\partial X^m}{\partial \sigma^\mu} - \overline{\theta}\Gamma^m \frac{\partial \theta}{\partial \sigma^\mu}$.

$$\mathcal{F}_{\mu\nu} = \frac{\partial A_\nu}{\partial \sigma^\mu} - \frac{\partial A_\mu}{\partial \sigma^\nu} + (\overline{\theta}\Gamma_{11}\Gamma_m \partial_\nu \theta)\left(\Pi_\mu^m + \frac{1}{2}\overline{\theta}\Gamma^m \partial_\mu \theta\right) - (\overline{\theta}\Gamma_{11}\Gamma_m \partial_\mu \theta)\left(\Pi_\nu^m + \frac{1}{2}\overline{\theta}\Gamma^m \partial_\nu \theta\right) \tag{1.3}$$

$$\theta_1 = \not{V}\theta, \qquad \theta_2 = \widetilde{\not{V}}\not{V}\theta \tag{1.4}$$

with

$$\not{V} = \{(\overline{\theta}\Gamma^m d\theta) + \Gamma_{11}(\overline{\theta}\Gamma_{11}\Gamma^m d\theta)\}\Gamma_m \tag{1.5}$$

$$\widetilde{\not{V}} = \{(\overline{\theta}\Gamma^m d\theta) - \Gamma_{11}(\overline{\theta}\Gamma_{11}\Gamma^m d\theta)\}\Gamma_m \tag{1.6}$$

$$\not{\Pi} = (dX^m + \overline{\theta}\Gamma^m d\theta)\Gamma_m \tag{1.7}$$

$$\mathcal{F} = \frac{1}{2}\mathcal{F}_{\mu\nu} d\sigma^\mu d\sigma^\nu \tag{1.8}$$

and $m, n = 0, .., 9/\mu, \nu = 0, 1, 2$.

The action (1.1) has a global supersymmetry and also a local supersymmetry (kappa symmetry).

The action for the non-relativistic D2-brane is obtained from (1.1) by doing the following rescaling [17]–[18]:

$$X^\mu = \omega x^\mu \tag{1.9}$$

$$X^a = x^a \tag{1.10}$$

$$T = \frac{1}{\omega} T_{NR} \tag{1.11}$$

$$A_i = \omega w_i \tag{1.12}$$

where the subindex $_{NR}$ stands for non-relativistic and $i, \mu = 0, 1, 2/a = 3, .., 9$.

$$\theta = \sqrt{\omega}\theta_- + \frac{1}{\sqrt{\omega}}\theta_+ \tag{1.13}$$

where θ_+, θ_- are eigenstates of

$$\Gamma_\star \equiv \Gamma_0 \Gamma_1 \Gamma_2 \tag{1.14}$$

$$\Gamma_\star \theta_\pm = \pm\theta_\pm \tag{1.15}$$

The action of the non-relativistic D2-brane is obtained by expanding (1.1) in powers of ω and keeping the finite part as $\omega \to \infty$ [18]

$$S_{NR} = \int d^3\sigma \mathcal{L}_{NR}^{DBI} + \int \mathcal{L}_{NR}^{WZ} \tag{1.16}$$

where

$$\begin{aligned}\mathcal{L}_{NR}^{DBI} = T_{NR}(\epsilon_{ijk} R_i^0 R_j^1 R_k^2) \Bigg[- \left(\overline{\theta}_+ \widehat{\gamma}^0 \frac{\partial \theta_+}{\partial \sigma^0} + \overline{\theta}_+ \widehat{\gamma}^1 \frac{\partial \theta_+}{\partial \sigma^1} + \overline{\theta}_+ \widehat{\gamma}^2 \frac{\partial \theta_+}{\partial \sigma 2} \right) + \\ \frac{1}{2} \widehat{g}^{il} (\eta_{a\acute{a}} u_i^a u_l^{\acute{a}}) + \frac{1}{4} \widetilde{\mathcal{F}}_{il}^{(1)} \widetilde{\mathcal{F}}_{jk}^{(1)} \widehat{g}^{ij} \widehat{g}^{lk} \Bigg]\end{aligned} \tag{1.17}$$

$$\begin{aligned}\mathcal{L}_{NR}^{WZ} = T_{NR} \Bigg[\frac{1}{2} (\overline{\theta}_+ \Gamma_{\mu\nu} d\theta_+) \Big((dx^\mu + \overline{\theta}_- \Gamma^\mu d\theta_-)(dx^\nu + \overline{\theta}_- \Gamma^\nu d\theta_-) \\ - (\overline{\theta}_- \Gamma^\mu d\theta_-)(dx^\nu + \frac{2}{3} \overline{\theta}_- \Gamma^\nu d\theta_-) \Big) + \frac{1}{2} (\overline{\theta}_- \Gamma_{\mu\nu} d\theta_-)(\overline{\theta}_+ \Gamma^\mu d\theta_+) \left(dx^\nu + \frac{2}{3} \overline{\theta}_- \Gamma^\nu d\theta_- \right) \\ + \frac{1}{2} (\overline{\theta}_- \Gamma_{ab} d\theta_-) \Big(\left(dx^a + \overline{\theta}_+ \Gamma^a d\theta_- + \overline{\theta}_- \Gamma^a d\theta_+ \right) (dx^b + \overline{\theta}_+ \Gamma^b d\theta_- + \overline{\theta}_- \Gamma^b d\theta_+) \\ - (\overline{\theta}_+ \Gamma^a d\theta_- + + \overline{\theta}_- \Gamma^a d\theta_+) \left(dx^b + \frac{2}{3} (\overline{\theta}_- \Gamma^b d\theta_+ + \overline{\theta}_+ \Gamma^b d\theta_-) \right) \Big) + \\ (\overline{\theta}_+ \Gamma_{\mu a} d\theta_- + \overline{\theta}_- \Gamma_{\mu a} d\theta_+) \left((dx^\mu + \overline{\theta}_- \Gamma^\mu d\theta_-) \left(dx^a + \frac{1}{2} (\overline{\theta}_+ \Gamma^a d\theta_- + \overline{\theta}_- \Gamma^a d\theta_+ \right) \right) \\ - \frac{1}{2} (\overline{\theta}_- \Gamma^\mu d\theta_-) \left(dx^a + \frac{1}{3} (\overline{\theta}_- \Gamma^a d\theta_+ + \overline{\theta}_+ \Gamma^a d\theta_-) \right) + \frac{1}{2} (\overline{\theta}_+ \Gamma_\mu \Gamma_{11} d\theta_- + \overline{\theta}_- \Gamma_\mu \Gamma_{11} d\theta_+) \\ \left(dx^\mu + \frac{2}{3} \overline{\theta}_- \Gamma^\mu d\theta_- \right) (\overline{\theta}_+ \Gamma_{11} d\theta_- + \overline{\theta}_- \Gamma_{11} d\theta_+) + \frac{1}{2} (\overline{\theta}_- \Gamma_b \Gamma_{11} d\theta_-) \Big(dx^b + \frac{2}{3} (\overline{\theta}_- \Gamma^b d\theta_+ + \\ \overline{\theta}_+ \Gamma^b d\theta_-) \Big) (\overline{\theta}_+ \Gamma_{11} d\theta_- + \overline{\theta}_- \Gamma_{11} d\theta_+) + (d\overline{\theta}_- \Gamma_{11} \theta_+ + d\overline{\theta}_+ \Gamma_{11} \theta_-)(f - (\overline{\theta}_- \Gamma_\mu \Gamma_{11} d\theta_+ + \\ \overline{\theta}_+ \Gamma_\mu \Gamma_{11} d\theta_-) \left(dx^\mu + \frac{1}{2} \overline{\theta}_- \Gamma^\mu d\theta_- \right) - (\overline{\theta}_- \Gamma_a \Gamma_{11} d\theta_-) \left(dx^a + \frac{1}{2} (\overline{\theta}_- \Gamma^a d\theta_+ + \overline{\theta}_+ \Gamma^a d\theta_-) \right) \Bigg]\end{aligned} \tag{1.18}$$

where

$$R_i^\mu = \partial_i x^\mu - \overline{\theta}_- \Gamma^\mu \partial_i \theta_- \tag{1.19}$$

$$u_i^a = \partial_i x^a - \overline{\theta}_- \Gamma^a \partial_i \theta_+ - \overline{\theta}_+ \Gamma^a \partial_i \theta_- \tag{1.20}$$

$i, \mu = 0, 1, 2/a = 3, .., 9$; also

$$\widehat{\gamma}^0 = \frac{1}{(\epsilon_{ijk} R_i^0 R_j^1 R_k^2)} (\epsilon_{ijk} \Gamma^i R_1^j R_2^k) \tag{1.21}$$

$$\widehat{\gamma}^1 = -\frac{1}{(\epsilon_{ijk} R_i^0 R_j^1 R_k^2)} (\epsilon_{ijk} \Gamma^i R_0^j R_2^k) \tag{1.22}$$

$$\widehat{\gamma}^2 = \frac{1}{(\epsilon_{ijk} R_i^0 R_j^1 R_k^2)} (\epsilon_{ijk} \Gamma^i R_0^j R_1^k) \tag{1.23}$$

$$\widehat{g}^{jk} = \eta_{\mu\nu} R_j^\mu R_k^\nu \tag{1.24}$$

where we have introduced the following quantities $\epsilon_{012} = 1$, $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1)$, $\eta_{a\acute{a}} = \mathrm{diag}(1, .., 1)$, $\mu, \nu = 0, 1, 2$; $a, \acute{a} = 3, .., 9$;

$$\begin{aligned} \widetilde{\mathcal{F}}_{ij}^{(1)} = f_{ij} &+ \left[(\overline{\theta}_- \Gamma_{11} \Gamma_\mu \partial_j \theta_+ + \overline{\theta}_+ \Gamma_{11} \Gamma_\mu \partial_j \theta_-) \left(R_i^\mu + \frac{1}{2} \overline{\theta}_- \Gamma^\mu \partial_i \theta_- \right) - (i \leftrightarrow j) \right] \\ &+ \left[(\overline{\theta}_- \Gamma_{11} \Gamma_a \partial_j \theta_-) \left(u_i^a + \frac{1}{2} (\overline{\theta}_- \Gamma^a \partial_i \theta_+ + \overline{\theta}_+ \Gamma^a \partial_i \theta_-) \right) - (i \leftrightarrow j) \right], \end{aligned} \tag{1.25}$$

where $f_{ij} = \partial_i w_j - \partial_j w_i$, $f = \frac{1}{2} f_{ij} d\sigma^i d\sigma^j$, and $\mu, \nu, i, j = 0, 1, 2/a = 3, .., 9$.

We denote by $[\mathcal{L}_{NR}^{WZ}]_3$ the 3-form coefficient of $\mathcal{L}_{NR}^{WZ}$ given in (1.18). The Lagrangian density of the non-relativistic D2-brane is given by:

$$\mathcal{L}_{NR} = \mathcal{L}_{NR}^{DBI} + [\mathcal{L}_{NR}^{WZ}]_3 \tag{1.26}$$

The action (1.16) is invariant under the non-relativistic counterpart of the global and local supersymmetric transformations that leave the relativistic D2-brane action (1.1) invariant [17].

The conjugate momenta of the variables $x^\mu, x^a, \theta_+, \theta_-, w_i$ are given by:

$$\begin{aligned} p_\mu = \frac{\partial \mathcal{L}_{NR}}{\partial \dot{x}^\mu} &= \\ \frac{\partial \mathcal{L}_{NR}}{\partial R_0^\mu} &+ \frac{\partial \mathcal{L}_{NR}}{\partial \widetilde{\mathcal{F}}_{0i}^{(1)}} (\overline{\theta}_- \Gamma_{11} \Gamma_\mu \partial_i \theta_+ + \overline{\theta}_+ \Gamma_{11} \Gamma_\mu \partial_i \theta_-) \end{aligned} \tag{1.27}$$

$\mu = 0, 1, 2/i = 1, 2$;

$$\begin{aligned} p_a = \frac{\partial \mathcal{L}_{NR}}{\partial \dot{x}^a} &= \\ \frac{\partial \mathcal{L}_{NR}}{\partial \dot{u}_0^a} &+ \frac{\partial \mathcal{L}_{NR}}{\partial \widetilde{\mathcal{F}}_{0i}^{(1)}} (\overline{\theta}_- \Gamma_{11} \Gamma_a \partial_i \theta_-) \end{aligned} \tag{1.28}$$

$a = 3, .., 9$;

$$\begin{aligned} J_{+l} = \frac{\partial^r \mathcal{L}_{NR}}{\partial \dot{\theta}_{+l}} &- \frac{\partial \mathcal{L}_{NR}}{\partial u_0^a} (\overline{\theta}_- \Gamma^a)_l - \\ \frac{\partial \mathcal{L}_{NR}}{\partial \widetilde{\mathcal{F}}_{0i}^{(1)}} &\left[\left(R_i^\mu + \frac{1}{2} \overline{\theta}_- \Gamma^\mu \partial_i \theta_- \right) (\overline{\theta}_- \Gamma_{11} \Gamma_\mu)_l + \frac{1}{2} (\overline{\theta}_- \Gamma_{11} \Gamma_a \partial_i \theta_-) (\overline{\theta}_- \Gamma^a)_l \right] \end{aligned} \tag{1.29}$$

and

$$J_{-l}=\frac{\partial^r\mathcal{L}_{NR}}{\partial\dot{\theta}_{-l}}=\frac{\partial^r[\mathcal{L}_{NR}^{WZ}]_3}{\partial\dot{\theta}_{-l}}-\frac{\partial\mathcal{L}_{NR}}{\partial R_0^\mu}(\overline{\theta}_-\Gamma^\mu)_l-$$

$$\frac{\partial\mathcal{L}_{NR}}{\partial u_0^a}(\overline{\theta}_+\Gamma^a)_l-\frac{\mathcal{L}_{NR}}{\partial\widetilde{\mathcal{F}}_{0i}^{(1)}}\left[\frac{1}{2}(\overline{\theta}_-\Gamma_{11}\Gamma_\mu\partial_i\theta_++\overline{\theta}_+\Gamma_{11}\Gamma_\mu\partial_i\theta_-)(\overline{\theta}_-\Gamma^\mu)_l+\right.$$

$$\frac{1}{2}(\overline{\theta}_-\Gamma_{11}\Gamma_a\partial_i\theta_-)(\overline{\theta}_+\Gamma^a)_l+\left(R_i^\mu+\frac{1}{2}\overline{\theta}_-\Gamma^\mu\partial_i\theta_-\right)(\overline{\theta}_+\Gamma_{11}\Gamma_\mu)_l+$$

$$\left.\left(u_i^a+\frac{1}{2}(\overline{\theta}_-\Gamma^a\partial_i\theta_++\overline{\theta}_+\Gamma^a\partial_i\theta_-)\right)(\overline{\theta}_-\Gamma_{11}\Gamma_a)_l\right], \tag{1.30}$$

where $l=1,..,32$; and

$$E^i=\frac{\partial\mathcal{L}_{NR}}{\partial\dot{w}_i}=\frac{\partial\mathcal{L}_{NR}}{\partial\widetilde{\mathcal{F}}_{0i}^{(1)}}\,. \tag{1.31}$$

The fermionic constraints are given by

$$F_{+l}=J_{+l}+p_a(\overline{\theta}_-\Gamma^a)_l+E^i\left[\left(R_i^\mu+\frac{1}{2}\overline{\theta}_-\Gamma^\mu\partial\theta_-\right)(\overline{\theta}_-\Gamma_{11}\Gamma_\mu)_l\right.$$

$$\left.-\frac{1}{2}(\overline{\theta}_-\Gamma_{11}\Gamma_a\partial_i\theta_-)(\overline{\theta}_-\Gamma^a)_l\right]+T_{NR}\epsilon_{ijk}(\overline{\theta}_+\Gamma^i)_lR_1^jR_2^k-\frac{\partial^r[\mathcal{L}_{NR}^{WZ}]_3}{\partial\dot{\theta}_{+l}} \tag{1.32}$$

where the last derivative does not include differentiation with respect to $u_0^a,\widetilde{\mathcal{F}}_{0i}^{(1)}$, and

$$F_{-l}=J_{-l}+p_a(\overline{\theta}_+\Gamma^a)_l+p_\mu(\overline{\theta}_-\Gamma^\mu)_l+E^i\left[\left(R_i^\mu+\frac{1}{2}(\overline{\theta}_-\Gamma^\mu\partial_i\theta_-\right)(\overline{\theta}_+\Gamma_{11}\Gamma_\mu)_l+\right.$$

$$\left(u_i^a+\frac{1}{2}(\overline{\theta}_-\Gamma^a\partial_i\theta_++\overline{\theta}_+\Gamma^a\partial_i\theta_-\right)(\overline{\theta}_-\Gamma_{11}\Gamma_a)_l-\frac{1}{2}(\overline{\theta}_-\Gamma_{11}\Gamma_\mu\partial_i\theta_++$$

$$\left.\overline{\theta}_+\Gamma_{11}\Gamma_\mu\partial_i\theta_-)(\overline{\theta}_-\Gamma^\mu)_l-\frac{1}{2}(\overline{\theta}_-\Gamma_{11}\Gamma_a\partial_i\theta_-)(\overline{\theta}_+\Gamma^a)_l\right]-\frac{\partial^r[\mathcal{L}_{NR}^{WZ}]_3}{\partial\dot{\theta}_{-l}}\,, \tag{1.33}$$

where the last derivative does not include differentiation with respect to u_0^a, R_0^μ, $\widetilde{\mathcal{F}}_{0i}^{(1)}$.

We introduce now two commuting spinor fields λ_+,λ_- corresponding to the fermionic constraints F_+,F_- and we write down a BRST charge according to the ansatz proposed in [1]–[4]:

$$
\begin{aligned}
Q = \int d^2\sigma (\lambda_+^l J_{+l} + \lambda_-^l J_{-l} + p_a(\overline{\theta}_-\Gamma^a\lambda_+ + \overline{\theta}_+\Gamma^a\lambda_-) + p_\mu(\overline{\theta}_-\Gamma^\mu\lambda_-) + \\
E^i\left[\left(R_i^\mu + \frac{1}{2}\overline{\theta}_-\Gamma^\mu\partial_i\theta_-\right)(\overline{\theta}_+\Gamma_{11}\Gamma_\mu\lambda_- + \overline{\theta}_-\Gamma_{11}\Gamma_\mu\lambda_+) + \Big(u_i^a + \frac{1}{2}(\overline{\theta}_-\Gamma^a\partial_i\theta_+ + \right. \\
\overline{\theta}_+\Gamma^a\partial_i\theta_-)\Big)(\overline{\theta}_-\Gamma_{11}\Gamma_a\lambda_-) - \frac{1}{2}(\overline{\theta}_-\Gamma_{11}\Gamma_\mu\partial_i\theta_+ + \overline{\theta}_+\Gamma_{11}\Gamma_\mu\partial_i\theta_-)(\overline{\theta}_-\Gamma^\mu\lambda_-) - \\
\left.\frac{1}{2}(\overline{\theta}_-\Gamma_{11}\Gamma_a\partial_i\theta_-)(\overline{\theta}_+\Gamma^a\lambda_- + \overline{\theta}_-\Gamma^a\lambda_+)\right] + T_{NR}\epsilon_{ijk}(\overline{\theta}_+\Gamma^i\lambda_+)R_1^jR_2^k - \left(\lambda_+^T\frac{\partial^r[\mathcal{L}_{NR}^{WZ}]_3}{\partial\dot{\theta}_+}\right) - \\
\left(\lambda_-^T\frac{\partial^r[\mathcal{L}_{NR}^{WZ}]_3}{\partial\dot{\theta}_-})\right) \quad (1.34)
\end{aligned}
$$

with $i, j, k, \mu = 0, 1, 2/a = 3, .., 9$.

A set of BRST transformations is the following:

$$
\begin{aligned}
s\theta_+^l &= \lambda_+^l, \\
s\theta_-^l &= \lambda_-^l, \\
sx^\mu &= \overline{\theta}_-\Gamma^\mu\lambda_-, \\
sx^a &= \overline{\theta}_-\Gamma^a\lambda_+ + \overline{\theta}_+\Gamma^a\lambda_-, \\
sR_\nu^\mu &= 2(\partial_\nu\overline{\theta}_-\Gamma^\mu\lambda_-), \\
su_\mu^a &= 2(\partial_\mu\overline{\theta}_-\Gamma^a\lambda_+ + \partial_\mu\overline{\theta}_+\Gamma^a\lambda_-), \\
s\widetilde{\mathcal{F}}_{ij}^{(1)} &= 2R_j^\mu(\overline{\lambda}_-\Gamma_{11}\Gamma_\mu\partial_i\theta_+ + \overline{\lambda} + \Gamma_{11}\Gamma_\mu\partial_i\theta_-) + 2u_j^a(\partial_i\overline{\theta}_-\Gamma_{11}\Gamma^a\lambda_-) - (i \leftrightarrow j)
\end{aligned} \quad (1.35)
$$

where again $\mu, \nu = 0, 1, 2/i, j = 1, 2/a = 3, .., 9/l = 1, ..32$; from which we obtain

$$
\begin{aligned}
s^2\theta_+^l &= s^2\theta_-^l = 0, \\
s^2x^\mu &= \overline{\lambda}_-\Gamma^\mu\lambda_-, \\
s^2x^a &= 2(\overline{\lambda}_-\Gamma^a\lambda_+), \\
s^2R_i^\mu &= \frac{\partial}{\partial\sigma^i}(\overline{\lambda}_-\Gamma^\mu\lambda_-), \\
s^2u_i^a &= 2\frac{\partial}{\partial\sigma^i}(\overline{\lambda}_+\Gamma^a\lambda_-)
\end{aligned} \quad (1.36)
$$

and

$$
\begin{aligned}
s^2\widetilde{\mathcal{F}}_{ij}^{(1)} = 2R_j^\mu\frac{\partial}{\partial\sigma^i}(\overline{\lambda}_+\Gamma_{11}\Gamma_\mu\lambda_-) + u_j^a\frac{\partial}{\partial\sigma^i}(\overline{\lambda}_-\Gamma_{11}\Gamma_a\lambda_-) + \\
2(\partial_j\overline{\theta}_-\Gamma_{11}\Gamma^\mu\partial_i\theta_-)(\overline{\lambda}_-\Gamma_\mu\lambda_-) + 2(\partial_j\overline{\theta}_-\Gamma^a\partial_i\theta_+)(\overline{\lambda}_-\Gamma_{11}\Gamma_a\lambda_-) + \\
2(\partial_j\overline{\theta}_-\Gamma_{11}\Gamma_a\partial_i\theta_-)(\overline{\lambda}_-\Gamma^a\lambda_+) + 2(\partial_j\theta_-\Gamma^\mu\partial_i\theta_-)(\overline{\lambda}_-\Gamma_{11}\Gamma_\mu\lambda_+)
\end{aligned} \quad (1.37)
$$

where we used the ansatz

$$\lambda_- = P_-\lambda,$$
$$P_- = \frac{1}{2}(1 - \Gamma_0\Gamma_1\Gamma_2) \tag{1.38}$$

The full set of constraints required for the nilpotency of the BRST charge is obtained by studying the equal time Poisson bracket $\{Q(\sigma), Q(\sigma')\}$ where $Q(\sigma)$ is given in (1.34). A basis for expanding a 32×32 matrix M is given in Appendix A. If we ignore the last two terms in (1.34) involving the derivatives of $[\mathcal{L}^{WZ}_{NR}]_3$ we obtain:

$$\begin{aligned}
\{Q(\sigma), Q(\sigma')\} = \int d^2\sigma[(\partial_i E^i)((\overline{\theta}_-\Gamma^a\lambda_+ + \overline{\theta}_+\Gamma^a\lambda_-)(\overline{\theta}_-\Gamma_{11}\Gamma_a\lambda_-) + \\
(\overline{\theta}_-\Gamma^\mu\lambda_-)(\overline{\theta}_+\Gamma_{11}\Gamma_\mu\lambda_- + \overline{\theta}_-\Gamma_{11}\Gamma_\mu\lambda_+)) + 4(\overline{\lambda}_+\Gamma^a\lambda_-)(p_a - E^i(\overline{\theta}_-\Gamma_{11}\Gamma_a\partial_i\theta_-)) + \\
2(\overline{\lambda}_-\Gamma^\mu\lambda_-)(p_\mu - E^i(\overline{\theta}_-\Gamma_{11}\Gamma_\mu\partial_i\theta_+ + \overline{\theta}_+\Gamma_{11}\Gamma_\mu\partial_i\theta_-)) + 4(\overline{\lambda}_+\Gamma_{11}\Gamma_\mu\lambda_-)E^iR^\mu_i + \\
2(\overline{\lambda}_-\Gamma_{11}\Gamma_a\lambda_-)E^iu^a_i + 2T_{NR}\epsilon_{\mu\nu\rho}(\overline{\lambda}_+\Gamma^\mu\lambda_+)R^\nu_1R^\rho_2 + 2E^i(-(\overline{\lambda}_-\Gamma_a\theta_-)(\partial_i\overline{\theta}_-\Gamma_{11}\Gamma^a\lambda_+ + \\
\partial_i\overline{\theta}_+\Gamma_{11}\Gamma^a\lambda_-) - (\overline{\theta}_-\Gamma_{11}\Gamma^\mu\lambda_-)(\partial_i\overline{\theta}_-\Gamma_\mu\lambda_+ + \partial_i\overline{\theta}_+\Gamma_\mu\lambda_-) + (\partial_i\overline{\theta}_-\Gamma_{11}\Gamma^\mu\lambda_-) \\
(\overline{\lambda}_-\Gamma_\mu\theta_+ + \overline{\theta}_-\Gamma_\mu\lambda_+) + (\overline{\lambda}_-\Gamma_a\partial_i\theta_-)(\overline{\theta}_+\Gamma_{11}\Gamma^a\lambda_- + \overline{\theta}_-\Gamma_{11}\Gamma^a\lambda_+))] + \\
4T_{NR}\epsilon_{0jk}\int d^2\sigma(\overline{\theta}\Gamma_{\nu\rho}\lambda_+)(\partial_j\overline{\theta}_-\Gamma^\nu\lambda_-)R^\rho_k,
\end{aligned} \tag{1.39}$$

where $\epsilon_{012} = 1$.

The final expression for the equal time Poisson bracket $\{Q(\sigma), Q(\sigma')\}$ where $Q(\sigma)$ is the full expression for the BRST charge given in (1.34) is given by:

$$\begin{aligned}
\{Q(\sigma), Q(\sigma')\} = \int d^2\sigma(\partial_i E^i)[(\overline{\theta}_-\Gamma^a\lambda_+ + \overline{\theta}_+\Gamma^a\lambda_-)(\overline{\theta}_-\Gamma_{11}\Gamma_a\lambda_-) + \\
(\overline{\theta}_-\Gamma^\mu\lambda_-)(\overline{\theta}_+\Gamma_{11}\Gamma_\mu\lambda_- + \overline{\theta}_-\Gamma_{11}\Gamma_\mu\lambda_+)] + \\
2T_{NR}\epsilon_{\mu\nu\rho}\int d^2\sigma R^\nu_1R^\rho_2(\overline{\lambda}_+\Gamma^\mu\lambda_+) + T_{NR}\epsilon_{0jk}\int d\sigma^2R^\mu_jR^\nu_k(\overline{\lambda}_+\Gamma_{\mu\nu}\lambda_+) + \\
\int d^2\sigma[F_\mu(\overline{\lambda}_-\Gamma^\mu\lambda_-) + F_a(\overline{\lambda}_-\Gamma^a\Gamma_{11}\lambda_-) + F_{ab}(\overline{\lambda}_-\Gamma^{ab}\lambda_-) + \\
F^{\nu abdf}(\overline{\lambda}_-\Gamma_{\nu abdf}\lambda_-) + \overline{F}(\overline{\lambda}_-\Gamma_{11}\lambda_+) + \overline{F}_a(\overline{\lambda}_-\Gamma^a\lambda_+) + \\
\overline{F}_{\nu b}(\overline{\lambda}_-\Gamma^{\nu b}\lambda_+) + \overline{F}_\nu(\overline{\lambda}_-\Gamma^\nu\Gamma_{11}\lambda_+) + \overline{F}_{\nu abc}(\overline{\lambda}_-\Gamma^{\nu abc}\lambda_+) + \overline{F}^{\nu ab}(\overline{\lambda}_+\Gamma_{\nu ab}\Gamma_{11}\lambda_-) + \\
\overline{F}_{fglmn}(\overline{\lambda}_+\Gamma^{fglmn}\lambda_-) + \overline{F}_{lmn}(\overline{\lambda}_-\Gamma^{lmn}\lambda_+)]
\end{aligned} \tag{1.40}$$

where now $i = 1, 2/\mu, \nu, \rho = 0, 1, 2/a, b, c, d, f, g, l, m, n = 3, .., 9;$ and the ansatz $\lambda_- = P_-\lambda$ is used.

The expressions for $F_\mu, F_a, F_{ab}, F_{\nu abdf}, \overline{F}, \overline{F}_a, \overline{F}_\nu, \overline{F}_{\nu b}, \overline{F}_{\nu abc}, \overline{F}_{\nu ab}, \overline{F}_{fglm}, \overline{F}_{lmn}$ are given in Appendix B. These expressions are different from zero, so the following constraints guarantee the nilpotency of the BRST charge:

$$
\begin{aligned}
&\partial_i E^i = 0 \qquad \text{(Gauss law)}, \\
&\overline{\lambda}_+ \Gamma_\mu \lambda_+ = 0, \qquad \overline{\lambda}_+ \Gamma_{\mu\nu} \lambda_+ = 0, \\
&\overline{\lambda}_- \Gamma_\mu \lambda_- = 0, \qquad \overline{\lambda}_- \Gamma_a \Gamma_{11} \lambda_- = 0, \qquad \overline{\lambda}_- \Gamma_{ab} \lambda_- = 0, \qquad \overline{\lambda}_- \Gamma_{\mu abcd} \lambda_- = 0, \\
&\overline{\lambda}_- \Gamma_{11} \lambda_+ = 0, \qquad \overline{\lambda}_- \Gamma_a \lambda_+ = 0, \qquad \overline{\lambda}_- \Gamma_{\mu a} \lambda_+ = 0, \qquad \overline{\lambda}_- \Gamma_\mu \Gamma_{11} \lambda_+ = 0, \\
&\overline{\lambda}_- \Gamma_{\mu abc} \lambda_+ = 0, \qquad \overline{\lambda}_+ \Gamma_{\mu ab} \Gamma_{11} \lambda_- = 0, \qquad \overline{\lambda}_+ \Gamma_{fglmn} \lambda_- = 0, \qquad \overline{\lambda}_- \Gamma_{lmn} \lambda_+ = 0
\end{aligned}
\tag{1.41}
$$

with the ansatz $\lambda_- = P_- \lambda, (i = 1, 2/\mu, \nu = 0, 1, 2/a, b, c, d, f, g, l, m, n = 3, .., 9)$.

2 Solving the constraints

In order to solve the constraints (1.41) we use the methodology used in [22]–[23]. The Dirac matrices Γ^m $(m = 0, 1, .., 9)$ are combined into five creation operators a^i $(i = 1, .., 5)$ and five annihilation operators a_i $(i = 1, .., 5)$ as follows:

$$
\begin{aligned}
&a^1 = \frac{1}{2}(\Gamma^1 - i\Gamma^2), \qquad a^2 = \frac{1}{2}(\Gamma^3 - i\Gamma^4), \qquad a^3 = \frac{1}{2}(\Gamma^5 - i\Gamma^6), \\
&a^4 = \frac{1}{2}(\Gamma^7 - i\Gamma^8), \qquad a^5 = \frac{1}{2}(\Gamma^9 + \Gamma^0), \\
&a_1 = \frac{1}{2}(\Gamma^1 + i\Gamma^2), \qquad a_2 = \frac{1}{2}(\Gamma^3 + i\Gamma^4), \qquad a_3 = \frac{1}{2}(\Gamma^5 + i\Gamma^6), \\
&a_4 = \frac{1}{2}(\Gamma^7 + i\Gamma^8), \qquad a_5 = \frac{1}{2}(\Gamma^9 - \Gamma^0)
\end{aligned}
\tag{2.1}
$$

The following identities hold:

$$
\{a_i, a^j\} = \delta_{ij}, \{a^i, a^j\} = 0, \qquad \{\Gamma_{11}, a^i\} = 0, \qquad \{\Gamma_{11}, a_j\} = 0, \qquad a_i = a^{i\dagger}, \qquad \text{with now} \quad i, j = 1, .., 5. \tag{2.2}
$$

Here we introduce the vacuum state $\mid 0 >$ and the state $< 0 \mid$ with the equations:

$$
a_i \mid 0 >= 0, \qquad < 0 \mid a^i = 0, \qquad i = 1, .., 5. \tag{2.3}
$$

We decompose a 32-component Dirac spinor into a sum of a positive chirality 16-component

part and a negative chirality 16-component part as follows:

$$
\begin{aligned}
\mid \lambda >= \left(\lambda_+ \mid 0> + \frac{1}{2}\lambda_{ij}a^j a^i \mid 0> + \frac{1}{4!}\lambda^i \epsilon_{ijklm} a^j a^k a^l a^m \mid 0>\right) + \\
\left(\frac{1}{5!}\lambda'_+ \epsilon_{ijklm} a^i a^j a^k a^l a^m \mid 0> + \frac{1}{3!}\lambda'^{ij}\epsilon_{ijklm} a^k a^l a^m \mid 0> + \lambda'_i a^i \mid 0>\right) = \\
(1+\overline{10}+5)+(1+10+\overline{5}),
\end{aligned}
\tag{2.4}
$$

where $\lambda_{ij} = -\lambda_{ji}, \lambda'^{ij} = -\lambda'^{ji}$.

Let us try to solve from (1.41) the constraints $\overline{\lambda}_+\Gamma_\mu\lambda_+ = 0$ and $\overline{\lambda}_+\Gamma_{\mu\nu}\lambda_+ = 0$ (where $\mu, \nu = 0, 1, 2$).

It is interesting to note that λ_+ can be written as a sum of two pure spinors $\lambda_{1+} = P_+\lambda_+$ and $\lambda_{2+} = P_-\lambda_+$ where $P_\pm = \frac{1}{2}(1 \pm \Gamma_0\Gamma_1\Gamma_2)$ and the constraints $\overline{\lambda}_+\Gamma_\mu\lambda_+ = 0$, $\overline{\lambda}_+\Gamma_{\mu\nu}\lambda_+ = 0, (\mu, \nu = 0, 1, 2)$ can be expressed equivalently as two pure spinor constraints: $\overline{\lambda}_{1+}\Gamma_m\lambda_{1+} = 0$,$\overline{\lambda}_{2+}\Gamma_m\lambda_{2+} = 0, (m = 0, ..., 9)$.

The constraints $\overline{\lambda}_+\Gamma_\mu\lambda_+ = 0$ can be written as follows:

$$
\begin{aligned}
< \lambda_+ \mid Ca_1 \mid \lambda_+ >= 0, \\
< \lambda_+ \mid Ca^1 \mid \lambda_+ >= 0, \\
< \lambda_+ \mid Ca_5 \mid \lambda_+ >=< \lambda_+ \mid Ca^5 \mid \lambda_+ > .
\end{aligned}
\tag{2.5}
$$

Let us suppose that $\mid \lambda_+ >$ is chiral:

$$
\mid \lambda_+ >= \lambda_{++} \mid 0> + \frac{1}{2}\lambda_{+ij}a^j a^i \mid 0> + \frac{1}{4!}\lambda_+^i \epsilon_{ijklm} a^j a^k a^l a^m \mid 0>, \tag{2.6}
$$

then we obtain

$$
\begin{aligned}
< \lambda_+ \mid Ca^1 \mid \lambda_+ >= 2\left(\lambda_{++}\lambda_+^1 + \frac{1}{8}\epsilon_{1ijkl}\lambda_{+ij}\lambda_{+lk}\right), \\
< \lambda_+ Ca^5 \mid \lambda_+ >= 2\left(\lambda_{++}\lambda_+^5 + \frac{1}{8}\epsilon_{5ijkl}\lambda_{+ij}\lambda_{+lk}\right), \\
< \lambda_+ \mid Ca_1 \mid \lambda_+ >= 2\lambda_+^i\lambda_{+i1}, \\
< \lambda_+ \mid Ca_5 \mid \lambda_+ >= 2\lambda_+^i\lambda_{+i5},
\end{aligned}
\tag{2.7}
$$

where now $\epsilon_{12345} = 1$.

The constraints $\overline{\lambda}_+\Gamma_{\mu\nu}\lambda_+ = 0, (\mu, \nu = 0, 1, 2)$ can be expressed as:

$$
\begin{aligned}
< \lambda_+ \mid C(a^5 - a_5)(a^1 + a_1) \mid \lambda_+ >= 0, \\
< \lambda_+ \mid C(a^1 + a_1)(a^1 - a_1) \mid \lambda_+ >= 0, \\
< \lambda_+ \mid C(a^5 - a_5)(a^1 - a_1) \mid \lambda_+ >= 0.
\end{aligned}
\tag{2.8}
$$

These expressions are identically zero for $\mid \lambda_+ >$ chiral, so from (2.5) and (2.7) we get

$$\begin{aligned} \lambda_+^1 &= \frac{1}{8\lambda_{++}}\epsilon_{1ijkl}\lambda_{+ij}\lambda_{+kl}, \\ \lambda_+^5 &= \frac{1}{\lambda_{++}}(\lambda_+^i\lambda_{+i5} + \frac{1}{8}\epsilon_{5ijkl}\lambda_{+ij}\lambda_{+kl}), \\ \lambda_+^i\lambda_{+i1} &= 0. \end{aligned} \tag{2.9}$$

For the general case we can write

$$\begin{aligned} &\mid \lambda_+ >= \lambda_{++} \mid 0> + \frac{1}{2}\lambda_{+ij}a^j a^i \mid 0> + \frac{1}{4!}\lambda_+^i\epsilon_{ijklm}a^j a^k a^l a^m \mid 0> + \\ &\lambda'_{+i}a^i \mid 0> + \frac{1}{3!}\lambda_+^{\prime ij}\epsilon_{ijlkm}a^l a^k a^m \mid 0> + \frac{1}{5!}\lambda'_{++}\epsilon_{ijklm}a^i a^j a^k a^l a^m \mid 0> . \end{aligned} \tag{2.10}$$

Thus we have the following relations:

$$< \lambda_+ \mid Ca^1 \mid \lambda_+ >= 2\left(\lambda_{++}\lambda_+^1 + \frac{1}{8}\epsilon_{1ijkl}\lambda_{+ij}\lambda_{+lk}\right) - 4\lambda'_{+i}\lambda_+^{\prime i1} = 0, \tag{2.11}$$

$$< \lambda_+ \mid Ca_1 \mid \lambda_+ >= 2\lambda^i\lambda_{+i1} - 2\lambda'_{+1}\lambda'_{++} + \epsilon_{1iji'j'}\lambda_+^{\prime ij}\lambda_+^{\prime i'j'} = 0, \tag{2.12}$$

$$\begin{aligned} &< \lambda_+ \mid Ca^5 \mid \lambda_+ >=< \lambda_+ \mid Ca_5 \mid \lambda_+ > \qquad \Rightarrow \\ &2(\lambda_{++}\lambda_+^5 + \frac{1}{8}\epsilon_{5ijkl}\lambda_{+ij}\lambda_{+lk}) - 4\lambda'_{+i}\lambda_+^{\prime i5} = 2\lambda_+^i\lambda_{+i5} - 2\lambda'_{+5}\lambda'_{++} + \epsilon_{5iji'j'}\lambda_+^{\prime ij}\lambda_+^{\prime i'j'}. \end{aligned} \tag{2.13}$$

On the other hand the constraints $\overline{\lambda}_+\Gamma^{\mu\nu}\lambda_+ = 0$ can be rewritten as follows:

$$\begin{aligned} &\frac{1}{2} < \lambda_+ \mid C(a_1 a^1 - a^1 a_1) \mid \lambda_+ >= \\ &-\lambda_{++}\lambda'_{++} + 4\lambda_{+i1}\lambda_+^{\prime i1} - \lambda_{+ij}\lambda^{\prime ij} + \lambda_+^i\lambda'_{+i} - 2\lambda_+^1\lambda'_{+1} = 0, \\ &< \lambda_+ \mid C(a^5 - a_5)a^1 \mid \lambda_+ >= \\ &4\lambda_{++}\lambda_+^{\prime 51} + 4\lambda_+^{\prime i1}\lambda_{+i5} + 2\lambda'_{+5}\lambda_+^1 - \epsilon_{15iji'}\lambda_{+ij}\lambda'_{+i'} = 0, \\ &\frac{1}{2} < \lambda_+ \mid C(a^5 - a_5)a_1 \mid \lambda_+ >= \\ &-2\lambda_{+i1}\lambda_+^{\prime i5} + \lambda_{+51}\lambda'_{++} + \lambda_+^5\lambda'_{+1} - \epsilon_{ijk15}\lambda_+^{\prime ij}\lambda_+^k = 0. \end{aligned} \tag{2.14}$$

From (2.11)-(2.14) we obtain

$$
\begin{aligned}
\lambda_{+}^{1} &= \frac{1}{\lambda_{++}}(2\lambda'_{+i}\lambda'^{i1}_{+} + \frac{1}{8}\epsilon_{1ijkl}\lambda_{+ij}\lambda_{+kl}), \\
\lambda'_{+1} &= \frac{1}{2\lambda'_{++}}(2\lambda^{i}_{+}\lambda_{+i1} + \epsilon_{1ijkl}\lambda'^{ij}_{+}\lambda'^{kl}_{+}), \\
\lambda^{5}_{+} &= \frac{1}{\lambda_{++}}\left(\lambda^{i}_{+}\lambda_{+i5} - \lambda'_{+5}\lambda'_{++} + 2\lambda'_{+i}\lambda'^{i5}_{+} + \frac{1}{2}\epsilon_{5ijkl}\lambda'^{ij}_{+}\lambda'^{kl}_{+} + \frac{1}{8}\epsilon_{5ijkl}\lambda_{+ij}\lambda_{+kl}\right), \\
\lambda'^{51}_{+} &= \frac{1}{4\lambda_{++}}(-4\lambda'^{i1}_{+}\lambda_{+i5} - 2\lambda'_{+5}\lambda^{1}_{+} + \epsilon_{15ijk}\lambda_{+ij}\lambda'_{+k}), \\
\lambda_{+51} &= \frac{1}{\lambda'_{++}}(2\lambda_{+i1}\lambda'^{i5}_{+} - \lambda^{5}_{+}\lambda'_{+1} + \epsilon_{15ijk}\lambda'^{ij}\lambda^{k}_{+}), \\
\lambda'_{++} &= \frac{1}{\lambda_{++}}(4\lambda_{+i1}\lambda'^{i1}_{+} - \lambda_{+ij}\lambda'^{ij}_{+} + \lambda^{i}_{+}\lambda'_{+i} - 2\lambda^{1}_{+}\lambda'_{+1}).
\end{aligned} \tag{2.15}
$$

Now from (1.41) we deal with the following constraints:

$$
\overline{\lambda}_{-}\Gamma^{\mu}\lambda_{-} = 0, \qquad \overline{\lambda}_{-}\Gamma^{a}\Gamma_{11}\lambda_{-} = 0, \qquad \overline{\lambda}_{-}\Gamma^{ab}\lambda_{-} = 0, \qquad \overline{\lambda}_{-}\Gamma^{\mu abcd}\lambda_{-} = 0, \tag{2.16}
$$

where $\mu = 0, 1, 2/a, b, c, d = 3, .., 9$ with $\lambda_{-} = P_{-}\lambda$ (P_{-} is given in (1.38)). Thus, we can write

$$
P_{-} = \frac{1}{2}[1 + i(a^{5} - a_{5})(a_{1}a^{1} - a^{1}a_{1})]. \tag{2.17}
$$

We have that $P_{-}^{T}C = CP_{-}$ where C is the charge conjugation matrix. So for example the constraints $\overline{\lambda}_{-}\Gamma^{\mu}\lambda_{-} = 0$ can be rewritten as follows:

$$
\begin{aligned}
&< \lambda \mid CP_{-}a^{1}P_{-} \mid \lambda >= 0, \\
&< \lambda \mid CP_{-}a_{1}P_{-} \mid \lambda >= 0, \\
&< \lambda \mid CP_{-}a^{5}P_{-} \mid \lambda >=< \lambda \mid CP_{-}a_{5}P_{-} \mid \lambda > .
\end{aligned} \tag{2.18}
$$

The solution of the constraints in (2.16) is the trivial one. This can be shown as follows. Let us consider the following 16 constraints:

$$
\begin{aligned}
&\overline{\lambda}_{-}\Gamma^{13579}\lambda_{-} = 0, \quad \overline{\lambda}_{-}\Gamma^{13589}\lambda_{-} = 0, \quad \overline{\lambda}_{-}\Gamma^{13679}\lambda_{-} = 0, \quad \overline{\lambda}_{-}\Gamma^{13689}\lambda_{-} = 0 \\
&\overline{\lambda}_{-}\Gamma^{14579}\lambda_{-} = 0, \quad \overline{\lambda}_{-}\Gamma^{14679}\lambda_{-} = 0, \quad \overline{\lambda}_{-}\Gamma^{14589}\lambda_{-} = 0, \overline{\lambda}_{-}\Gamma^{14689}\lambda_{-} = 0 \\
&\overline{\lambda}_{-}\Gamma^{23579}\lambda_{-} = 0, \quad \overline{\lambda}_{-}\Gamma^{23589}\lambda_{-} = 0, \quad \overline{\lambda}_{-}\Gamma^{23679}\lambda_{-} = 0, \overline{\lambda}_{-}\Gamma^{23689}\lambda_{-} = 0, \\
&\overline{\lambda}_{-}\Gamma^{24579}\lambda_{-} = 0, \quad \overline{\lambda}_{-}\Gamma^{24589}\lambda_{-} = 0, \quad \overline{\lambda}_{-}\Gamma^{24679}\lambda_{-} = 0, \overline{\lambda}_{-}\Gamma^{24689}\lambda_{-} = 0.
\end{aligned} \tag{2.19}
$$

which can be rewritten as:

$$
\begin{aligned}
&< \lambda \mid Ca^{1}((a^{5} + a_{5}) - i(a_{5}a^{5} - a^{5}a_{5}))a^{2}a^{3}a^{4} \mid \lambda >= \\
&\qquad -(\lambda_{+})^{2} + (\lambda'_{5})^{2} + 2i(\lambda_{+}\lambda'_{5}) = 0
\end{aligned} \tag{2.20}
$$

$$< \lambda \mid Ca^1((a^5+a_5)-i(a_5a^5-a^5a_5))a^2a^3a_4 \mid \lambda >=$$
$$-(\lambda_{45})^2+(\lambda_4')^2-2i(\lambda_4'\lambda_{45})=0 \tag{2.21}$$

$$< \lambda \mid Ca^1((a^5+a_5)-i(a_5a^5-a^5a_5))a^2a_3a^4 \mid \lambda >=$$
$$-(\lambda_{35})^2+(\lambda_3')^2-2i(\lambda_3'\lambda_{35})=0 \tag{2.22}$$

$$< \lambda \mid Ca^1((a^5+a_5)-i(a_5a^5-a^5a_5))a_2a^3a^4 \mid \lambda >=$$
$$-(\lambda_{52})^2+(\lambda_2')^2-2i(\lambda_2'\lambda_{52})=0 \tag{2.23}$$

$$< \lambda \mid Ca^1((a^5+a_5)-i(a_5a^5-a^5a_5))a^2a_3a_4 \mid \lambda >=$$
$$-(\lambda_{34})^2+4(\lambda^{'12})^2-4i(\lambda_{34}\lambda^{'12})=0 \tag{2.24}$$

$$< \lambda \mid Ca^1((a^5+a_5)-i(a_5a^5-a^5a_5))a_2a^3a_4 \mid \lambda >=$$
$$-(\lambda_{42})^2+4(\lambda^{'13})^2-4i(\lambda_{42}\lambda^{'13})=0 \tag{2.25}$$

$$< \lambda \mid Ca^1((a^5+a_5)-i(a_5a^5-a^5a_5))a_2a_3a^4 \mid \lambda >=$$
$$-(\lambda_{23})^2+4(\lambda^{'14})^2-4i(\lambda_{23}\lambda^{'14})=0 \tag{2.26}$$

$$< \lambda \mid Ca^1((a^5+a_5)-i(a_5a^5-a^5a_5))a_2a_3a_4 \mid \lambda >=$$
$$-(\lambda^1)^2+4(\lambda^{'15})^2-4i(\lambda^1\lambda^{'15})=0 \tag{2.27}$$

$$< \lambda \mid Ca_1((a^5+a_5)-i(a_5a^5-a^5a_5))a^2a^3a^4 \mid \lambda >=$$
$$-(\lambda_{15})^2+(\lambda_1')^2+2i(\lambda_1'\lambda_{15})=0 \tag{2.28}$$

$$< \lambda \mid Ca_1((a^5+a_5)-i(a_5a^5-a^5a_5))a^2a^3a_4 \mid \lambda >=$$
$$-(\lambda_{14})^2+4(\lambda^{'23})^2+4i(\lambda_{14}\lambda^{'23})=0 \tag{2.29}$$

$$< \lambda \mid Ca_1((a^5+a_5)-i(a_5a^5-a^5a_5))a^2a_3a^4 \mid \lambda >=$$
$$-(\lambda_{13})^2+4(\lambda^{'24})^2-4i(\lambda_{13}\lambda^{'24})=0 \tag{2.30}$$

$$< \lambda \mid Ca_1((a^5+a_5)-i(a_5a^5-a^5a_5))a_2a^3a^4 \mid \lambda >=$$
$$-(\lambda_{12})^2+4(\lambda^{'34})^2+4i(\lambda_{12}\lambda^{'34})=0 \tag{2.31}$$

$$< \lambda \mid Ca_1((a^5+a_5) - i(a_5a^5 - a^5a_5))a^2a_3a_4 \mid \lambda >= \\ -(\lambda^2)^2 + 4(\lambda'^{52})^2 - 4i(\lambda^2\lambda'^{52}) = 0 \tag{2.32}$$

$$< \lambda \mid Ca_1((a^5+a_5) - i(a_5a^5 - a^5a_5))a_2a^3a_4 \mid \lambda >= \\ -(\lambda^3)^2 + 4(\lambda'^{53})^2 + 4i(\lambda^3\lambda'^{53}) = 0 \tag{2.33}$$

$$< \lambda \mid Ca_1((a^5+a_5) - i(a_5a^5 - a^5a_5))a_2a_3a^4 \mid \lambda >= \\ -(\lambda^4)^2 + 4(\lambda'^{45})^2 + 4i(\lambda^4\lambda'^{45}) = 0 \tag{2.34}$$

$$< \lambda \mid Ca_1((a^5+a_5) - i(a_5a^5 - a^5a_5))a_2a_3a_4 \mid \lambda >= \\ -(\lambda^5)^2 + (\lambda'_+)^2 - 2i(\lambda^5\lambda'_+) = 0 \tag{2.35}$$

and the expression for $\mid \lambda >$ is given in (2.4). We follow the claim made in [22]–[23] that a chiral spinor can be chosen to be real and assume that $\mid \lambda >$ is real. Then the relations (2.20)-(2.35) imply that all the components of $\mid \lambda >$ are zero as a solution to the constraints (2.16). It is interesting to note that λ_- corresponds to θ_- which is a gauge degree of freedom as pointed out in [17].

3 Conclusions

In this paper we treated the non-relativistic IIA D2 brane in the framework of the pure spinor formalism [24]. We derived the fermionic constraints corresponding to the rescaled fermionic coordinates. We introduced two commuting spinor fields each one corresponding to a fermionic coordinate. The nilpotency of the BRST charge leads to a set of constraints for the two spinor fields including pure spinor constraints. Nontrivial solutions are found for the spinor field λ_+ which corresponds to the fermionic coordinate θ_+. It is interesting to note that the solution for the spinor field λ_- corresponding to θ_- which, according to the proof given in [17] constitutes a gauge degree of freedom, is the trivial one. Thus, we can set $\theta- = 0$ without loss of generality. This study can be performed for more general manifolds and for 11 dimensions as well. We would like to finally mention that the treatment of the relativistic Dp-brane in the framework of pure spinor formalism has been reported in [25].

Acknowledgments

We gratefully acknowledge support from "Programa PAPIIT, UNAM, IN103413-3, *Teorías de Kaluza-Klein, inflación y perturbaciones gravitacionales.* AHA acknowledges a VIEP-BUAP-HEAA-EXC15-I research grant; he thanks as well SNI and PRODEP for partial financial support, while JEP is grateful to the Instituto de Física y Matemáticas at Morelia for the warm hospitality and the stimulating atmosphere generated during his visit at the Institute.

4 Appendix A

For the matrices $\Gamma^m\,(m=0,..,9)$ we use the Majorana representation (Γ^0 is real antisymmetric, $\Gamma^i\,(i=1,..,9)$ are real symmetric)

$$\{\Gamma^m,\Gamma^n\}=2\eta^{mn} \tag{4.1}$$

where $\eta^{mn}=diag(-1,1,..,1)$ and $m,n=0,..,9$; and

$$\begin{aligned} C &= \Gamma_0,\\ (\Gamma^m)^T &= -\Gamma_0\Gamma^m\Gamma_0^{-1}\,. \end{aligned} \tag{4.2}$$

A basis for the 32×32 matrices is given by (see Appendix B in [26] for instance):

$$B=\left\{I,\Gamma^A\Gamma^{AB},\Gamma^{ABC},\Gamma^{ABCD},\Gamma^{ABCDE},\Gamma^{ABCD}\Gamma_{11},\Gamma^{AB}\Gamma_{11},\Gamma^A\Gamma_{11},\Gamma_{11}\right\}, \tag{4.3}$$

with $A<B<C<D<E$.

For the differential form $d\theta$ we use the convention given in [19]:

$$d\theta=d\sigma^\mu\partial_\mu\theta=-\partial_\mu\theta d\sigma^\mu\,. \tag{4.4}$$

The following identities hold for $\lambda_-=P_-\lambda$:

$$\begin{aligned} \Gamma_{\mu\nu}\lambda_- &= -\epsilon_{\mu\nu\rho}\Gamma^\rho\lambda_-\\ \Gamma_{\mu\nu\rho}\lambda_- &= -\epsilon_{\mu\nu\rho}\lambda_- \end{aligned} \tag{4.5}$$

$\epsilon_{012}=1$.

5 Appendix B

$$
\begin{aligned}
F_\mu = 2[p_\mu - E^i(\overline{\theta}_-\Gamma_{11}\Gamma_\mu\partial_i\theta_+ + \overline{\theta}_+\Gamma_{11}\Gamma_\mu\partial_i\theta_-)] + 2T_{NR}\epsilon_{0jk}[-2(\overline{\theta}_+\Gamma_{\mu\nu}\partial_j\theta_+)R^\nu_k + \\
(\overline{\theta}_+\Gamma_{a\mu}\partial_j\theta_- + \overline{\theta}_-\Gamma_{a\mu}\partial_j\theta_+)u^a_k] + \frac{T_{NR}}{96}\epsilon_{0jk}[(\overline{\theta}_-\Gamma^\nu\partial_k\theta_-)(-89(\overline{\theta}_+\Gamma_{\mu\nu}\partial_j\theta_+) + \\
22(\overline{\theta}_+\partial_j\theta_+)\eta_{\mu\nu}) + (\overline{\theta}_+\Gamma^\nu\partial_k\theta_+)(83(\overline{\theta}_-\Gamma_{\mu\nu}\partial_j\theta_-) + 6(\overline{\theta}_-\partial_j\theta_-)\eta_{\mu\nu}) - 2(\overline{\theta}_-\Gamma^a\partial_k\theta_+ + \\
\overline{\theta}_+\Gamma^a\partial_k\theta_-)(47(\overline{\theta}_+\Gamma_{\mu a}\partial_j\theta_-) + 23(\overline{\theta}_-\Gamma_{\mu a}\partial_j\theta_+)) - 5(\overline{\theta}_+\Gamma_\mu\Gamma_{\nu a}\partial_j\theta_-)(\partial_k\overline{\theta}_+\Gamma^{\nu a}\theta_-) - \\
\frac{1}{3}(\partial_k\overline{\theta}_+\Gamma^a\theta_-)(37(\partial_j\overline{\theta}_-\Gamma_{\mu a}\theta_+) - 66(\partial_j\overline{\theta}_+\Gamma_{\mu a}\theta_-)) - (\partial_j\theta_-\Gamma^a\theta_+) \\
(21(\partial_k\overline{\theta}_+\Gamma_{\mu a}\theta_-) - 2(\partial_k\theta_-\Gamma_{\mu a}\theta_+)) - 2(\overline{\theta}_+\Gamma_{11}\partial\theta_- + \overline{\theta}_-\Gamma_{11}\partial\theta_+) \\
(25(\overline{\theta}_+\Gamma_{11}\Gamma_\mu\partial_j\theta_-) + 31(\overline{\theta}_-\Gamma_{11}\Gamma_\mu\partial_j\theta_+)) + 2(\overline{\theta}_-\Gamma_{11}\Gamma^\nu\partial_k\theta_+ + \overline{\theta}_+\Gamma_{11}\Gamma^\nu\partial_k\theta_-) \\
(\overline{\theta}_+\Gamma_{11}\Gamma_\mu\Gamma_\nu\partial_j\theta_- + 2(\overline{\theta}_-\Gamma_{11}\partial_j\theta_+)\eta_{\mu\nu}) - (\partial_j\overline{\theta}_-\Gamma_{11}\Gamma^\nu\theta_-)(64(\partial_k\overline{\theta}_+\Gamma_{11}\theta_-) - \\
11(\partial_k\overline{\theta}_+\Gamma_{11}\Gamma_{\mu\nu}\theta_-) + 46(\partial_k\overline{\theta}_-\Gamma_{11}\theta_+)\eta_{\mu\nu} - 2(\partial_k\overline{\theta}_-\Gamma_{11}\Gamma_\nu\Gamma_\mu\theta_+)) + (\overline{\theta}_-\Gamma_{11}\Gamma_\nu\partial_k\theta_+) \\
(9(\overline{\theta}_-\Gamma_{11}\Gamma_\nu\Gamma_\mu\theta_+) + 34(\overline{\theta}_-\Gamma_{11}\partial_j\theta_+)\eta_{\mu\nu} + 23(\overline{\theta}_+\Gamma_{11}\partial_j\theta_-)\eta_{\mu\nu}) - 7(\partial_j\overline{\theta}_-\Gamma_{11}\Gamma_a\theta_-) \\
(\partial_k\overline{\theta}_+\Gamma_{11}\Gamma_{\mu a}\theta_+) + 5(\partial_k\overline{\theta}_+\Gamma_{11}\Gamma^{\nu a}\Gamma_\mu\theta_+)(\overline{\theta}_-\Gamma_{\nu a}\Gamma_{11}\partial_j\theta_-) + (\overline{\theta}_-\Gamma^{\nu a}\Gamma_{11}\Gamma_\mu\partial_j\theta_-) \\
(\partial_k\overline{\theta}_+\Gamma_{\nu a}\Gamma_{11}\theta_+) - (\partial_k\overline{\theta}_+\Gamma_{11}\Gamma^a\theta_+)(\partial_j\overline{\theta}_-\Gamma_{11}\Gamma_{\mu a}\theta_-) + (\overline{\theta}_+\Gamma_{\nu a}\partial_j\theta_-)(\overline{\theta}_-\Gamma^{\nu a}\Gamma_\mu\partial_k\theta_+)]
\end{aligned}
\tag{5.1}
$$

$$
\begin{aligned}
F_a = -2E_iu^i_a + 2T_{NR}\epsilon_{0jk}(\overline{\theta}_+\Gamma_{11}\partial_j\theta_- + \overline{\theta}_-\Gamma_{11}\partial_j\theta_+)u_{ak} + \frac{T_{NR}}{3}\epsilon_{0jk}[-\frac{1}{16}(\overline{\theta}_+\Gamma^b\partial_j\theta_-) \\
(10(\overline{\theta}_-\Gamma_{ab}\Gamma_{11}\partial_k\theta_+) - (\overline{\theta}_+\Gamma_{ab}\Gamma_{11}\partial_k\theta_-) + 3(\partial_k\theta_-\Gamma_{11}\theta_+)\eta_{ab} + 45(\partial_k\overline{\theta}_+\Gamma_{11}\theta_-)\eta_{ab}) + \\
\frac{3}{16}(\overline{\theta}_-\Gamma^b\partial_j\theta_+)(-8(\partial_k\overline{\theta}_+\Gamma_{ab}\Gamma_{11}\theta_-) + 12(\overline{\theta}_-\Gamma_{11}\partial_k\theta_+)\eta_{ab} - (\overline{\theta}_+\Gamma_{11}\partial_k\theta_-)\eta_{ab}) + \\
\frac{7}{8}(\overline{\theta}_+\Gamma^\mu\partial_k\theta_+)(\partial_j\overline{\theta}_-\Gamma_{\mu a}\Gamma_{11}\theta_-) + \frac{1}{48}(\overline{\theta}_+\Gamma_{11}\Gamma^\mu\partial_k\theta_-)(5(\partial_j\overline{\theta}_+\Gamma_{\mu a}\theta_-) - 6(\partial_j\overline{\theta}_-\Gamma_{\mu a}\theta_+)) - \\
\frac{1}{16}(\partial_j\overline{\theta}_-\Gamma_{11}\Gamma^b\theta_-)(13(\partial_k\overline{\theta}_+\Gamma_{ab}\theta_+) + 3(\overline{\theta}_+\partial_k\theta_+)\eta_{ab}) - \frac{1}{32}(\partial_j\overline{\theta}_-\Gamma_{abc}\theta_+) \\
(2(\partial_k\overline{\theta}_+\Gamma^{bc}\Gamma_{11}\theta_-) + \partial_k\overline{\theta}_-\Gamma^{bc}\Gamma_{11}\theta_+) + \frac{1}{16}(\partial_j\overline{\theta}_-\Gamma^{\mu b}\theta_+)(-2(\partial_k\overline{\theta}_-\Gamma_{\mu ab}\Gamma_{11}\theta_+) \\
-6(\partial_k\overline{\theta}_+\Gamma_{11}\Gamma_{\mu ab}\theta_-) + 9(\overline{\theta}_-\Gamma_{11}\Gamma_\mu\partial_k\theta_+)\eta_{ab} + (\overline{\theta}_+\Gamma_{11}\Gamma_\mu\partial_k\theta_-)\eta_{ab}) - \frac{9}{16}(\overline{\theta}_-\Gamma^\mu\partial_j\theta_-) \\
(\overline{\theta}_+\Gamma_{\mu a}\Gamma_{11}\partial_k\theta_+) + \frac{3}{8}(\overline{\theta}_+\Gamma^{\mu b}\Gamma_{11}\partial_k\theta_+)(\partial_j\overline{\theta}_-\Gamma_{\mu ab}\theta_-) + \frac{117}{64}(\overline{\theta}_-\partial_j\theta_-)(\overline{\theta}_+\Gamma_{11}\Gamma_a\partial_k\theta_+) + \\
\frac{137}{64}(\overline{\theta}_-\Gamma_{ab}\partial_j\theta_-)(\overline{\theta}_+\Gamma_{11}\Gamma^b\partial_k\theta_+) + \frac{1}{16}(\overline{\theta}_-\Gamma^{\mu b}\partial_j\theta_+)(\partial_k\overline{\theta}_-\Gamma_{11}\Gamma_{\mu ab}\theta_+) - \frac{1}{16}(\overline{\theta}_-\Gamma_{11}\Gamma^{\mu b}\partial_j\theta_-) \\
(\partial_k\overline{\theta}_+\Gamma_{\mu ab}\theta_+) - \frac{1}{16}(\overline{\theta}_-\Gamma^{bc}\partial_k\theta_-)(\overline{\theta}_+\Gamma_{abc}\Gamma_{11}\partial_j\theta_+)]
\end{aligned}
\tag{5.2}
$$

$$
\begin{aligned}
F_{ab} = T_{NR}\epsilon_{0jk}u_{aj}u_{bk} + \frac{T_{NR}}{16}\epsilon_{0jk}[(\overline{\theta}_+\Gamma^d\partial_j\theta_-)(-\frac{14}{3}(\partial_k\overline{\theta}_-\Gamma_{ab}\Gamma_d\theta_+) + \\
\frac{14}{3}(\partial_k\overline{\theta}_-\Gamma_d\Gamma_{ab}\theta_+) + \frac{35}{6}(\partial_k\overline{\theta}_+\Gamma_{ab}\Gamma_d\theta_-) - \frac{15}{2}(\partial_k\overline{\theta}_+\Gamma_d\Gamma_{ab}\theta_-) + \frac{16}{3}(\overline{\theta}_+\Gamma_b\partial_k\theta_-)\eta_{ad} + \\
\frac{32}{3}(\overline{\theta}_-\Gamma_b\partial_k\theta_+)\eta_{ad}) + \frac{1}{6}(\overline{\theta}_-\Gamma^d\partial_j\theta_+)(7(\partial_k\overline{\theta}_+\Gamma_d\Gamma_{ab}\theta_-) + 35(\partial_k\overline{\theta}_+\Gamma_{ab}\Gamma_d\theta_-) + \\
4(\partial_k\overline{\theta}_-\Gamma_d\Gamma_{ab}\theta_+) - 33(\partial_k\overline{\theta}_-\Gamma_{ab}\Gamma_d\theta_+) - 32(\overline{\theta}_-\Gamma_a\partial_k\theta_+)\eta_{bd}) + \frac{1}{6}(\overline{\theta}_-\Gamma_{11}\Gamma^\mu\partial_k\theta_+) \\
(-4(\partial_j\overline{\theta}_+\Gamma_{11}\Gamma_{\mu ab}\theta_-) + 37(\partial_j\overline{\theta}_-\Gamma_{11}\Gamma_{\mu ab}\theta_-)) + \frac{15}{4}(\overline{\theta}_+\Gamma^{\mu d}\Gamma_{11}\partial_k\theta_+)(\partial_j\overline{\theta}_-\Gamma_{11}\Gamma_{\mu d}\Gamma_{ab}\theta_-) + \\
\frac{25}{12}(\partial_j\overline{\theta}_-\Gamma^{\mu d}\theta_+)(\partial_k\overline{\theta}_+\Gamma_{\mu d}\Gamma_{ab}\theta_-) - \frac{77}{12}(\overline{\theta}_+\Gamma^d\Gamma_{11}\partial_k\theta_+)(\partial_j\overline{\theta}_-\Gamma_{11}\Gamma_d\Gamma_{ab}\theta_-) - \\
\frac{35}{4}(\partial_j\overline{\theta}_-\Gamma_{11}\theta_+)(\partial_k\overline{\theta}_+\Gamma_{11}\Gamma_{ab}\theta_-) - \frac{45}{4}(\overline{\theta}_-\Gamma_{ab}\partial_j\theta_-)(\overline{\theta}_+\partial_k\theta_+) + \frac{37}{4}(\overline{\theta}_+\Gamma^\mu\partial_k\theta_+) \\
(\partial_j\overline{\theta}_-\Gamma_{\mu ab}\theta_-) + \frac{1}{6}(\overline{\theta}_-\Gamma_{11}\Gamma^d\partial_j\theta_-)(5(\partial_k\overline{\theta}_+\Gamma_{11}\Gamma_{ab}\Gamma_d\theta_+) + 39(\partial_k\overline{\theta}_+\Gamma_d\Gamma_{ab}\Gamma_{11}\theta_+)) - \\
3(\overline{\theta}_-\Gamma_{11}\partial_k\theta_+)(\partial_j\overline{\theta}_-\Gamma_{11}\Gamma_{ab}\theta_+) + (\overline{\theta}_-\Gamma^{\mu d}\partial_j\theta_+)(\partial_k\overline{\theta}_-\Gamma_{ab}\Gamma_{\mu d}\theta_+) - (\overline{\theta}_-\Gamma_{df}\partial_k\theta_-) \\
(\overline{\theta}_+\Gamma^d\Gamma_{ab}\Gamma^f\partial_j\theta_+) - \frac{1}{4}(\overline{\theta}_+\Gamma_{11}\Gamma^\mu\partial_k\theta_-)(\partial_j\overline{\theta}_+\Gamma_{11}\Gamma_{\mu ab}\theta_-) - \frac{1}{2}(\overline{\theta}_-\partial_j\theta_-)(\partial_k\overline{\theta}_+\Gamma_{ab}\theta_+) - \\
\frac{1}{6}(\overline{\theta}_-\Gamma_{11}\Gamma^{\mu d}\partial_j\theta_-)(\partial_k\overline{\theta}_+\Gamma_{11}\Gamma_{\mu ab}\Gamma_d\theta_+) + \frac{1}{6}(\partial_k\overline{\theta}_+\Gamma_{df}\Gamma_{11}\theta_-)(\partial_j\overline{\theta}_-\Gamma_{11}\Gamma^d\Gamma_{ab}\Gamma^f\theta_+) - \\
4(\overline{\theta}_-\Gamma^\mu\partial_j\theta_-)(\partial_k\overline{\theta}_+\Gamma_{\mu ab}\theta_+)]
\end{aligned} \tag{5.3}
$$

$$
\begin{aligned}
\overline{F} = 2T_{NR}\epsilon_{0jk}\Big[\widetilde{\mathcal{F}}^{(1)}_{jk} - \frac{1}{3}((\overline{\theta}_-\Gamma^\mu\partial_k\theta_-)(\overline{\theta}_+\Gamma_\mu\Gamma_{11}\partial_j\theta_- + \overline{\theta}_-\Gamma_\mu\Gamma_{11}\partial_j\theta_+) + \\
(\overline{\theta}_-\Gamma^a\Gamma_{11}\partial_j\theta_-)(\overline{\theta}_-\Gamma_a\partial_k\theta_+ + \overline{\theta}_+\Gamma_a\partial_k\theta_-))\Big]
\end{aligned} \tag{5.4}
$$

$$
\begin{aligned}
\overline{F}_\nu = -4E^iR_{\nu i} + 4T_{NR}\epsilon_{0jk}R_{\nu k}(\overline{\theta}_+\Gamma_{11}\partial_j\theta_- + \overline{\theta}-\Gamma_{11}\partial_j\theta_+) + \frac{1}{48}T_{NR}\epsilon_{0jk}[-(\overline{\theta}_-\Gamma^{\mu a}\partial_j\theta_+) \\
(16(\partial_k\overline{\theta}_-\Gamma_{11}\Gamma_a\theta_-)\eta_{\mu\nu} - 5(\partial_k\overline{\theta}_-\Gamma_{\mu\nu a}\Gamma_{11}\theta_-)) - 12(\partial_k\overline{\theta}_-\Gamma_{11}\Gamma^a\theta_-)(\overline{\theta}_+\Gamma_{\nu a}\partial_j\theta_-) - \\
(\overline{\theta}_-\Gamma^\mu\partial_j\theta_-)(-37(\partial_k\overline{\theta}_+\Gamma_{\mu\nu}\Gamma_{11}\theta_-) + 41(\partial_k\overline{\theta}_+\Gamma_{11}\theta_-)\eta_{\mu\nu} + 18(\partial_k\overline{\theta}_-\Gamma_{11}\theta_+)\eta_{\mu\nu}) + \\
(\partial_j\overline{\theta}_-\Gamma_{\nu a}\Gamma_b\theta_-)(-5(\partial_k\overline{\theta}_+\Gamma^b\Gamma^a\Gamma_{11}\theta_-) + 2(\partial_k\overline{\theta}_-\Gamma^b\Gamma^a\Gamma_{11}\theta_+)) - (\overline{\theta}_-\Gamma^{ab}\partial_k\theta_-) \\
(7(\overline{\theta}_-\Gamma_{\nu ab}\Gamma_{11}\partial_j\theta_+) + 2(\overline{\theta}_+\Gamma_{\nu ab}\Gamma_{11}\partial_j\theta_-)) + (\partial_j\overline{\theta}_+\Gamma_{11}\Gamma^\mu\theta_-)(70(\overline{\theta}_-\partial_k\theta_-)\eta_{\mu\nu} + \\
59(\partial_k\overline{\theta}_-\Gamma_{\mu\nu}\theta_-)) + (\partial_j\overline{\theta}_-\Gamma_{\nu a}\Gamma_{11}\theta_-)(49(\overline{\theta}_-\Gamma^a\partial_k\theta_+) + 24(\overline{\theta}_+\Gamma^a\partial_k\theta_-)) - \\
5(\partial_k\overline{\theta}_+\Gamma^{\mu a}\Gamma_\nu\theta_-)(\overline{\theta}_-\Gamma_{11}\Gamma_{\mu a}\partial_j\theta_-)]
\end{aligned} \tag{5.5}
$$

$$\begin{aligned}\overline{F}_a &= 4(p_a - E^i(\overline{\theta}_-\Gamma_{11}\Gamma_a\partial_i\theta_-)) - 2T_{NR}\epsilon_{0jk}[(\overline{\theta}_-\Gamma_{\mu a}\partial_k\theta_+ + \overline{\theta}_+\Gamma_{\mu a}\partial_k\theta_-)R^\mu_j \\ &-(\overline{\theta}_-\Gamma_{ab}\partial_k\theta_-)u^b_j] + \frac{2}{3}T_{NR}\epsilon_{0jk}[\frac{1}{16}(\overline{\theta}_-\Gamma^\mu\partial_j\theta_-)(25(\partial_k\overline{\theta}_+\Gamma_{\mu a}\theta_-) + 19(\partial_k\theta_-\Gamma_{\mu a}\theta_+)) + \\ &\frac{1}{8}(\overline{\theta}_-\partial_j\theta_-)(21(\partial_k\overline{\theta}_+\Gamma_a\theta_-) + 9(\partial_k\overline{\theta}_-\Gamma_a\theta_+)) - \frac{1}{32}(\partial_j\overline{\theta}_-\Gamma_{\mu ab}\theta_-)(5(\overline{\theta}_-\Gamma^{\mu b}\partial_k\theta_+) - \\ &2(\overline{\theta}_+\Gamma^{\mu b}\partial_k\theta_-)) + \frac{1}{32}(\overline{\theta}_-\Gamma_{ab}\partial_k\theta_-)(7(\overline{\theta}_-\Gamma^b\partial_j\theta_+) + 12(\overline{\theta}_+\Gamma^b\partial_j\theta_-)) + \frac{1}{16}(\partial_j\overline{\theta}_-\Gamma_{11}\Gamma_{\mu a}\theta_-) \\ &(3(\partial_k\overline{\theta}_-\Gamma_{11}\Gamma^\mu\theta_+) + 10(\partial_k\overline{\theta}_+\Gamma_{11}\Gamma^\mu\theta_-)) + \frac{1}{16}(\overline{\theta}_-\Gamma_a\Gamma_{11}\partial_j\theta_-)(29(\overline{\theta}_+\Gamma_{11}\partial_k\theta_-) + \\ &20(\overline{\theta}_-\Gamma_{11}\partial_k\theta_+)) + \frac{1}{32}(\overline{\theta}_-\Gamma_{11}\Gamma^b\partial_k\theta_-)(2(\overline{\theta}_+\Gamma_{11}\Gamma_{ab}\partial_j\theta_-) - \overline{\theta}_-\Gamma_{11}\Gamma^{ab}\partial_j\theta_+) + \\ &\frac{1}{32}(\partial_j\overline{\theta}_-\Gamma_{11}\Gamma_{abc}\theta_-)(\overline{\theta}_+\Gamma_{11}\Gamma^{bc}\partial_k\theta_- - 5(\overline{\theta}_-\Gamma_{11}\Gamma^{bc}\partial_k\theta_+)) + \frac{1}{32}(\overline{\theta}_-\Gamma^{bc}\partial_j\theta_-) \\ &(7(\partial_k\overline{\theta}_+\Gamma_{abc}\theta_-) + 2(\partial_k\overline{\theta}_-\Gamma_{abc}\theta_+)) - \frac{5}{32}(\partial_k\overline{\theta}_+\Gamma_{11}\Gamma_{\mu ab}\theta_-)(\overline{\theta}_-\Gamma_{11}\Gamma^{\mu b}\partial_j\theta_-)] \end{aligned} \tag{5.6}$$

$$\begin{aligned}\overline{F}_{\nu a} &= 4T_{NR}\epsilon_{0jk}R_{\nu j}u_{ak} + \frac{1}{48}T_{NR}\epsilon_{0jk}[(\overline{\theta}_-\Gamma^\mu\partial_j\theta_+)(37(\overline{\theta}_-\Gamma_a\partial_k\theta_+)\eta_{\mu\nu} + \\ &34(\overline{\theta}_+\Gamma_a\partial_k\theta_-)\eta_{\mu\nu} - 5(\overline{\theta}_-\Gamma_{\mu\nu a}\partial_k\theta_+) + 6(\overline{\theta}_+\Gamma_{\mu\nu a}\partial_k\theta_-)) + (\overline{\theta}_-\Gamma_{11}\Gamma^b\partial_j\theta_-) \\ &(5(\partial_k\overline{\theta}_+\Gamma_{11}\Gamma_\nu(\eta_{ab} + \Gamma_{ba})\theta_-) - 2(\partial_k\overline{\theta}_-\Gamma_{11}\Gamma_{\nu ba}\theta_+)) - (\overline{\theta}_-\Gamma^b\partial_k\theta_+) \\ &(-19(\partial_j\overline{\theta}_-\Gamma_{\nu ba}\theta_-) + 9(\partial_j\overline{\theta}_-\Gamma_\nu\theta_-)\eta_{ab}) + (\overline{\theta}_-\Gamma^{b\mu}\Gamma_{11}\partial_j\theta_-)(\partial_k\overline{\theta}_+\Gamma_\mu\Gamma_{11}\Gamma_{\nu a}\Gamma_b\theta_-) + \\ &5(\overline{\theta}_-\Gamma^\mu\Gamma_{11}\partial_k\theta_+)(\partial_j\overline{\theta}_-\Gamma_\mu\Gamma_{11}\Gamma_{\nu a}\theta_-) - (\overline{\theta}_-\Gamma^{bc}\Gamma_{11}\partial_k\theta_+)(\partial_j\overline{\theta}_-\Gamma_{11}\Gamma_{\nu abc}\theta_-) + \\ &(\partial_k\overline{\theta}_+\Gamma^{b\mu}\theta_-)(\partial_j\overline{\theta}_-\Gamma_\mu\Gamma_{\nu a}\Gamma_b\theta_-) - 5(\overline{\theta}_-\Gamma_{11}\partial_k\theta_+)(\overline{\theta}_-\Gamma_{\nu a}\Gamma_{11}\partial_j\theta_-) - 22(\overline{\theta}_+\Gamma^b\partial_j\theta_-) \\ &(\overline{\theta}_-\Gamma_{\nu ba}\partial_k\theta_-) + 2(\overline{\theta}_+\Gamma_{11}\Gamma^\mu\partial_k\theta_-)(-\partial_j\overline{\theta}_-\Gamma_{11}\Gamma_{\mu\nu a}\theta_- + 2(\partial_j\overline{\theta}_-\Gamma_{11}\Gamma_a\theta_-)\eta_{\mu\nu}) + \\ &2(\partial_k\overline{\theta}_-\Gamma^{b\mu}\theta_+)(2(\partial_j\overline{\theta}_-\Gamma_{\mu\nu}\theta_-)\eta_{ab} + (\partial_j\overline{\theta}_-\Gamma_{\mu\nu ba}\theta_-) - 2(\partial_j\overline{\theta}_-\Gamma_{ba}\theta_-)\eta_{\mu\nu} - \\ &(\partial_j\overline{\theta}_-\theta_-)\eta_{\mu\nu}\eta_{ab}) - 2(\partial_j\overline{\theta}_-\Gamma_{11}\theta_+)(\partial_k\overline{\theta}_-\Gamma_{11}\Gamma_{\nu a}\theta_-) + (\partial_k\overline{\theta}_-\Gamma^{bc}\theta_-) \\ &(\partial_j\overline{\theta}_-\Gamma_{\nu abc}\theta_+ - 2(\partial_j\overline{\theta}_+\Gamma_{\nu abc}\theta_-))] \end{aligned} \tag{5.7}$$

$$\begin{aligned}\overline{F}^{\nu ab} &= \frac{1}{96}T_{NR}\epsilon_{0jk}[(\partial_k\overline{\theta}_-\Gamma^\mu\Gamma_{11}\Gamma^{\nu ab}\Gamma^d\theta_-)(4(\partial_j\overline{\theta}_-\Gamma_{d\mu}\theta_+) - \partial_j\overline{\theta}_+\Gamma_{d\mu}\theta_-) - \\ &6(\partial_k\overline{\theta}_-\Gamma^d\theta_+)(\partial_j\overline{\theta}_-(\Gamma_d\Gamma^{\nu ab} + 3\Gamma^{\nu ab}\Gamma_d)\Gamma_{11}\theta_-) + 3(\overline{\theta}_-\Gamma^d\partial_k\theta_+) \\ &(\partial_j\overline{\theta}_-(\Gamma_d\Gamma^{\nu ab} + 6\Gamma^{\nu ab}\Gamma_d)\Gamma_{11}\theta_-) + (\partial_k\overline{\theta}_+\Gamma_{dc}\Gamma_{11}\theta_-)(\partial_j\overline{\theta}_-\Gamma^d\Gamma^{\nu ab}\Gamma^c\theta_-) - \\ &3(\partial_k\overline{\theta}_+\Gamma_{11}\theta_-)(\partial_j\overline{\theta}_-\Gamma^{\nu ab}\theta_-) + 3(\partial_j\overline{\theta}_-\Gamma^\mu\theta_-)(\partial_k\overline{\theta}_+\Gamma_\mu\Gamma_{11}\Gamma^{\nu ab}\theta_-) - \\ &(\overline{\theta}_-\Gamma_{cd}\partial_j\theta_-)(\partial_k\overline{\theta}_+\Gamma^c\Gamma_{11}\Gamma^{\nu ab}\Gamma^d\theta_-) + (\overline{\theta}_-\Gamma_{d\mu}\Gamma_{11}\partial_j\theta_-)(\partial_k\overline{\theta}_+\Gamma^\mu\Gamma^{\nu ab}\Gamma^d\theta_-) + \\ &3(\overline{\theta}_-\Gamma^\mu\Gamma_{11}\partial_k\theta_+)(\partial_j\overline{\theta}_-\Gamma_\mu\Gamma^{\nu ab}\theta_-) + 3(\overline{\theta}_-\partial_j\theta_-)(\partial_k\overline{\theta}_+\Gamma_{11}\Gamma^{\nu ab}\theta_-) - \\ &3(\overline{\theta}_-\Gamma_{11}\Gamma^d\partial_j\theta_-)(\overline{\theta}_-\Gamma^{\nu ab}\Gamma_d\partial_k\theta_-)] \end{aligned} \tag{5.8}$$

$$
\begin{aligned}
\overline{F}^{lmn} &= \frac{1}{48}T_{NR}\epsilon_{0jk}[-\frac{1}{4}(\overline{\theta}_+\Gamma^d\partial_k\theta_-)(\overline{\theta}_-\Gamma^{lmn}\Gamma_d\partial_j\theta_- + \overline{\theta}_-\Gamma_d\Gamma^{lmn}\partial_j\theta_-) - \\
&\frac{1}{32}(\overline{\theta}_-\Gamma^d\partial_k\theta_+)(5(\overline{\theta}_-\Gamma^{lmn}\Gamma_d\partial_j\theta_-) + 6(\overline{\theta}_-\Gamma_d\Gamma^{lmn}\partial_j\theta_-)) + \frac{1}{32}(\overline{\theta}_-\Gamma_\mu\partial_j\theta_-) \\
&(\partial_k\overline{\theta}_+\Gamma^{\mu lmn}\theta_-) + \frac{1}{32}(\overline{\theta}_-\Gamma_{\mu d}\partial_j\theta_+)(\overline{\theta}_-\Gamma^d\Gamma^{\mu lmn}\partial_k\theta_-) + \frac{1}{32}(\partial_k\overline{\theta}_+\Gamma_{11}\Gamma_\mu\theta_-) \\
&(\partial_j\overline{\theta}_-\Gamma_{11}\Gamma^{\mu lmn}\theta_-) - \frac{1}{32}(\overline{\theta}_-\Gamma_{11}\Gamma_d\partial_k\theta_-)(\overline{\theta}_-\Gamma_{11}\Gamma^{lmnd}\partial_j\theta_+) + \frac{1}{32}(\overline{\theta}_-\Gamma_{11}\partial_k\theta_+) \\
&(\overline{\theta}_-\Gamma^{lmn}\Gamma_{11}\partial_j\theta_-) - \frac{1}{32}(\partial_k\overline{\theta}_+\Gamma^{df}\Gamma_{11}\theta_-)(\partial_j\overline{\theta}_-\Gamma_{11}\Gamma_d\Gamma^{lmn}\Gamma_f\theta_-) + \frac{1}{16}(\partial_j\overline{\theta}_-\Gamma^{df}\Gamma_{11}\theta_+) \\
&(\partial_k\overline{\theta}_-\Gamma_{11}\Gamma_d\Gamma^{lmn}\Gamma_f\theta_-) - \frac{3}{32}(\overline{\theta}_-\Gamma_{11}\Gamma^l\partial_k\theta_-)(\overline{\theta}_-\Gamma_{11}\Gamma^{mn}\partial_j\theta_+ - 4(\overline{\theta}_+\Gamma_{11}\Gamma^{mn}\partial_j\theta_-)) - \\
&\frac{1}{32}((\overline{\theta}_-\Gamma_{11}\Gamma^{d\mu}\partial_j\theta_-)(\partial_k\overline{\theta}_+\Gamma_{11}\Gamma^{lmn}\Gamma_{d\mu}\theta_-) + (\overline{\theta}_-\Gamma_{df}\partial_j\theta_-)(\partial_k\overline{\theta}_+\Gamma^d\Gamma^{lmn}\Gamma^f\theta_-) + \\
&(\overline{\theta}_-\partial_j\theta_-)(\partial_k\overline{\theta}_+\Gamma^{lmn}\theta_-)) - \frac{1}{16}(\overline{\theta}_-\Gamma^{df}\partial_j\theta_-)(\overline{\theta}_+\Gamma_d\Gamma^{lmn}\Gamma_f\partial_k\theta_-)] - \\
&\frac{1}{96\times 4!\times 5!}\epsilon_{defglmn}\delta^{[abcd]}_{defg}[-5(\overline{\theta}_-\partial_j\theta_-)(\partial_k\overline{\theta}_+\Gamma_{11}\Gamma^{abcd}\theta_-) + 3(\partial_j\overline{\theta}_-\Gamma_\mu\theta_-) \\
&(\overline{\theta}_-\Gamma_{11}\Gamma^{\mu abcd}\partial_k\theta_+) + (\partial_j\theta_-\Gamma_{11}\Gamma^f\theta_-)(7(\overline{\theta}_-\Gamma^{abcd}\Gamma_f\partial_k\theta_+) - 2(\overline{\theta}_+(\Gamma^{abcd}\Gamma_f + \\
&\Gamma_f\Gamma^{abcd})\partial_k\theta_-)) - (\partial_k\overline{\theta}_+\Gamma_f\theta_-)(\overline{\theta}_-\Gamma^{abcd}\Gamma_{11}\Gamma^f\partial_j\theta_- + 6(\overline{\theta}_-\Gamma^f\Gamma^{abcd}\Gamma_{11}\partial_j\theta_-)) - \\
&8(\partial_k\overline{\theta}_-\Gamma_f\theta_+)(\overline{\theta}_-(\Gamma^{abcd}\Gamma^f + \Gamma^f\Gamma^{abcd})\Gamma_{11}\partial_j\theta_-) + (\overline{\theta}_-\Gamma_{\mu f}\partial_j\theta_+)(\partial_k\overline{\theta}_-\Gamma_{11}\Gamma^{abcd}\Gamma^{\mu f}\theta_- + \\
&2(\partial_k\overline{\theta}_-\Gamma^{\mu f}\Gamma^{abcd}\Gamma_{11}\theta_-)) - (\overline{\theta}_-\Gamma_{11}\Gamma_{\mu f}\partial_j\theta_-)(\partial_k\overline{\theta}_+\Gamma^{abcd}\Gamma^{\mu f}\theta_- + 2(\partial_k\overline{\theta}_+\Gamma^{\mu f}\Gamma^{abcd}\theta_-)) + \\
&(\partial_j\overline{\theta}_-\Gamma^f\Gamma^{abcd}\Gamma^e\theta_-)(\partial_k\overline{\theta}_+\Gamma_{fe}\Gamma_{11}\theta_- + 2(\partial_k\overline{\theta}_-\Gamma_{fe}\Gamma_{11}\theta_+)) - 5(\overline{\theta}_-\Gamma^{abcd}\partial_j\theta_-) \\
&(\overline{\theta}_-\Gamma_{11}\partial_k\theta_+) - (\overline{\theta}_-\Gamma_{ef}\partial_k\theta_-)(\overline{\theta}_-\Gamma^e\Gamma^{abcd}\Gamma^f\partial_j\theta_+ + 2(\overline{\theta}_+\Gamma^e\Gamma^{abcd}\Gamma^f\Gamma_{11}\partial_j\theta_-)) + \\
&3(\partial_j\overline{\theta}_-\Gamma^{\mu abcd}\theta_-)(\partial_k\overline{\theta}_+\Gamma_{11}\Gamma_\mu\theta_-)]
\end{aligned} \tag{5.9}
$$

$$
\begin{aligned}
\overline{F}_{\nu abc} &= \frac{1}{288}T_{NR}\epsilon_{0jk}[(\overline{\theta}_-\Gamma^\mu\partial_j\theta_-)(3(\overline{\theta}_-\Gamma_{\mu\nu abc}\partial_k\theta_+) - 23(\overline{\theta}_-\Gamma_{abc}\partial_k\theta_+)\eta_{\mu\nu}) + \\
&(\overline{\theta}_-\Gamma^d\partial_j\theta_+)(5(\overline{\theta}_-\Gamma_{\nu abcd}\partial_k\theta_-) + 3(\overline{\theta}_-\Gamma_{\nu bc}\partial_k\theta_-)\eta_{ad}) - 24(\overline{\theta}_+\Gamma_a\partial_j\theta_-) \\
&(\overline{\theta}_-\Gamma_{\nu bc}\partial_k\theta_-) + (\overline{\theta}_-\Gamma_{11}\Gamma^\mu\partial_k\theta_+)(-3(\overline{\theta}_-\Gamma_{11}\Gamma_{abc}\partial_j\theta_-)\eta_{\mu\nu} + 5(\partial_j\overline{\theta}_-\Gamma_{11}\Gamma_{\mu\nu abc}\theta_-)) + \\
&(\overline{\theta}_-\Gamma_{11}\Gamma^d\partial_j\theta_-)(\overline{\theta}_-\Gamma_{11}\Gamma_{\nu abc}\Gamma_d\partial_k\theta_+ - 4(\overline{\theta}_-\Gamma_d\Gamma_{\nu abc}\Gamma_{11}\partial_k\theta_+) + \\
&12(\overline{\theta}_+\Gamma_{\nu bc}\Gamma_{11}\partial_k\theta_-)\eta_{ad}) + (\overline{\theta}_-\Gamma^{\mu d}\partial_k\theta_+)(\partial_j\overline{\theta}_-\Gamma_\mu\Gamma_{\nu abc}\Gamma_d\theta_- - 4(\partial_j\overline{\theta}_-\Gamma_d\Gamma_{\nu abc}\Gamma_\mu\theta_-)) - \\
&(\overline{\theta}_-\Gamma^{df}\partial_k\theta_-)(\overline{\theta}_-\Gamma_d\Gamma_{\nu abc}\Gamma_f\partial_j\theta_+ + 2(\overline{\theta}_+\Gamma_d\Gamma_{\nu abc}\Gamma_f\partial_j\theta_-)) + (\partial_k\overline{\theta}_-\Gamma_d\Gamma_{11}\Gamma_{\nu abc}\Gamma_f\theta_-) \\
&(3(\partial_j\overline{\theta}_+\Gamma^{df}\Gamma_{11}\theta_-) + 2(\partial_j\overline{\theta}_-\Gamma^{df}\Gamma_{11}\theta_+)) + 5(\partial_j\overline{\theta}_+\Gamma_{11}\theta_-)(\partial_k\overline{\theta}_-\Gamma_{11}\Gamma_{\nu abc}\theta_-) + \\
&3(\overline{\theta}_-\Gamma_{11}\Gamma_{\mu d}\partial_j\theta_-)(\partial_k\overline{\theta}_+\Gamma^\mu\Gamma_{11}\Gamma_{\nu abc}\Gamma^d\theta_-) - 3(\overline{\theta}_-\partial_j\theta_-)(\partial_k\overline{\theta}_+\Gamma_{\nu abc}\theta_-)]
\end{aligned} \tag{5.10}
$$

$$
\begin{aligned}
F^{\nu abdf} = \frac{1}{96} T_{NR}\epsilon_{0jk}[&6(\overline{\theta}_-\Gamma^c\partial_j\theta_+)(\overline{\theta}_-\Gamma^{\nu abdf}\Gamma_c\partial_k\theta_+ - \overline{\theta}_-\Gamma_c\Gamma^{\nu abdf}\partial_k\theta_+) - \\
&2(\overline{\theta}_-\Gamma_{11}\Gamma_\mu\partial_k\theta_+)(\partial_j\overline{\theta}_+\Gamma_{11}\Gamma^\mu\Gamma^{\nu abdf}\theta_- + \partial_j\overline{\theta}_+\Gamma_{11}\Gamma^{\nu abdf}\Gamma^\mu\theta_-) - 2(\overline{\theta}_-\Gamma_{11}\Gamma_{\rho\sigma}\partial_k\theta_+) \\
&(\partial_j\overline{\theta}_+\Gamma_{11}\Gamma^\rho\Gamma^{\nu abdf}\Gamma^\sigma\theta_-) + 2(\overline{\theta}_-\Gamma^c\partial_j\theta_+)(\partial_k\overline{\theta}_-\Gamma^{\nu adf}\Gamma_c\theta_+ + \partial_k\overline{\theta}_-\Gamma_c\Gamma^{\nu abdf}\theta_+) + \\
&3(\overline{\theta}_+\Gamma^c\partial_j\theta_-)(-2(\partial_k\overline{\theta}_+\Gamma^{\nu abdf}\Gamma_c\theta_-) + \overline{\theta}_-\Gamma^{\nu abdf}\Gamma_c\partial_k\theta_+) - 4(\overline{\theta}_-\Gamma_{11}\Gamma_\mu\partial_k\theta_+) \\
&(\partial_j\overline{\theta}_-\Gamma_{11}\Gamma^\mu\Gamma^{\nu abdf}\theta_+) + (\overline{\theta}_+\Gamma_{11}\Gamma_\mu\partial_k\theta_-)(\partial_j\overline{\theta}_+\Gamma_{11}\Gamma^\mu\Gamma^{\nu abdf}\theta_- - \\
&2(\partial_j\overline{\theta}_+\Gamma_{11}\Gamma^{\nu abdf}\Gamma^\mu\theta_-)) - 2(\partial_j\overline{\theta}_-\Gamma^\mu\theta_-)(\partial_k\overline{\theta}_+\Gamma_\mu\Gamma^{\nu abdf}\theta_+) + 2(\partial_k\overline{\theta}_+\Gamma_{\mu c}\theta_-) \\
&(\overline{\theta}_+\Gamma^c\Gamma^{\nu abdf}\Gamma^\mu\partial_j\theta_-) + 2(\overline{\theta}_-\Gamma^c\Gamma_{11}\partial_j\theta_-)(\partial_k\overline{\theta}_+\Gamma_{11}\Gamma^{\nu abdf}\Gamma_c\theta_+ - \\
&2(\partial_k\overline{\theta}_+\Gamma_{11}\Gamma_c\Gamma^{\nu abdf}\theta_+)) + 2(\partial_k\overline{\theta}_+\Gamma_{11}\theta_-)(\overline{\theta}_+\Gamma_{11}\Gamma^{\nu abdf}\partial_j\theta_-) + (\overline{\theta}_+\Gamma^\mu\partial_k\theta_+) \\
&(6(\partial_j\overline{\theta}_-\Gamma_\mu\Gamma^{\nu abdf}\theta_-) + 5(\partial_j\theta_-\Gamma^{\nu abdf}\Gamma_\mu\theta_-)) + 7(\overline{\theta}_+\Gamma_{\mu\sigma}\partial_k\theta_+)(\partial_j\overline{\theta}_-\Gamma^\sigma\Gamma^{\nu abdf}\Gamma^\mu\theta_-) - \\
&3(\overline{\theta}_+\Gamma_{\mu c}\partial_j\theta_-)(\partial_k\overline{\theta}_+\Gamma^c\Gamma^{\nu abdf}\Gamma^\mu\theta_-) - 4(\overline{\theta}_+\Gamma_{11}\partial_j\theta_-)(\partial_k\overline{\theta}_+\Gamma_{11}\Gamma^{\nu abdf}\theta_-) + \\
&(\partial_k\overline{\theta}_+\Gamma_{\mu c}\Gamma_{11}\theta_+)(\partial_j\overline{\theta}_-\Gamma^c\Gamma_{11}\Gamma^{\nu abdf}\Gamma^\mu\theta_-) + (\partial_k\overline{\theta}_+\Gamma_\mu\Gamma_\sigma\theta_+) \\
&(\partial_j\overline{\theta}_-\Gamma^\sigma\Gamma^{\nu abdf}\Gamma^\mu\theta_-) + (\overline{\theta}_+\Gamma_{11}\Gamma_\sigma\Gamma_\mu\partial_j\theta_-)(\partial_k\overline{\theta}_+\Gamma^\sigma\Gamma_{11}\Gamma^{\nu abdf}\Gamma^\mu\theta_-) - \\
&(\partial_k\overline{\theta}_+\Gamma_{11}\Gamma_c\theta_+)(\partial_j\overline{\theta}_-\Gamma_{11}\Gamma^c\Gamma^{\nu abdf}\theta_-) + 2(\overline{\theta}_-\Gamma_{ce}\partial_k\theta_-)(\overline{\theta}_+\Gamma^c\Gamma^{\nu abdf}\Gamma^e\partial_j\theta_+) - \\
&\frac{1}{4!}\epsilon_{cgeabdf}\delta^{[lmn]}_{cge}\frac{T_{NR}}{576}[-36(\overline{\theta}_-\Gamma^l\partial_j\theta_+)(\partial_k\overline{\theta}_+\Gamma^{\nu mn}\Gamma_{11}\theta_-) + 2(\overline{\theta}_-\Gamma^r\partial_j\theta_+) \\
&(\partial_k\overline{\theta}_-\Gamma_{11}\Gamma^{\nu lmn}\Gamma_r\theta_+ + \partial_k\overline{\theta}_-\Gamma_{11}\Gamma_r\Gamma^{\nu lmn}\theta_+) + 3(\overline{\theta}_+\Gamma^r\partial_j\theta_-) \\
&(\partial_k\overline{\theta}_+\Gamma_r\Gamma^{\nu lmn}\Gamma_{11}\theta_- + 2(\partial_k\overline{\theta}_+\Gamma_{11}\Gamma^{\nu lmn}\Gamma_r\theta_-)) - 4(\overline{\theta}_-\Gamma_{11}\Gamma_\mu\partial_k\theta_+) \\
&(\partial_j\overline{\theta}_-\Gamma^\mu\Gamma^{\nu lmn}\theta_+) - (\overline{\theta}_+\Gamma_{11}\Gamma_\mu\partial_k\theta_-)(2(\partial_j\overline{\theta}_+\Gamma^{\nu lmn}\Gamma^\mu\theta_-) + \partial_j\overline{\theta}_+\Gamma^\mu\Gamma^{\nu lmn}\theta_-) + \\
&2(\overline{\theta}_-\Gamma^\mu\partial_j\theta_-)(\partial_k\overline{\theta}_+\Gamma_{11}\Gamma_\mu\Gamma^{\nu lmn}\theta_+) - 2(\partial_k\overline{\theta}_+\Gamma_{r\mu}\theta_-)(\overline{\theta}_+\Gamma^r\Gamma^{\nu lmn}\Gamma_{11}\Gamma^\mu\partial_j\theta_-) + \\
&2(\overline{\theta}_-\Gamma^r\Gamma_{11}\partial_j\theta_-)(-2(\partial_k\overline{\theta}_+\Gamma_r\Gamma^{\nu lmn}\theta_+) + \partial_k\overline{\theta}_+\Gamma^{\nu lmn}\Gamma_r\theta_+) + 2(\partial_k\overline{\theta}_+\Gamma_{11}\theta_-) \\
&(\overline{\theta}_+\Gamma^{\nu lmn}\partial_j\theta_-) + 6(\overline{\theta}_+\Gamma_{\sigma\mu}\partial_k\theta_+)(\partial_j\overline{\theta}_-\Gamma^\mu\Gamma^{\nu lmn}\Gamma_{11}\Gamma^\sigma\theta_-) - (\partial_k\overline{\theta}_+\theta_+) \\
&(\partial_j\overline{\theta}_-\Gamma^{\nu lmn}\Gamma_{11}\theta_-) + (\partial_k\overline{\theta}_+\Gamma_{\mu r}\Gamma_{11}\theta_+)(\partial_j\overline{\theta}_-\Gamma^r\Gamma^{\nu lmn}\Gamma^\mu\theta_-) + (\partial_j\overline{\theta}_-\Gamma_{11}\Gamma_{\mu\sigma}\theta_+) \\
&(\partial_k\overline{\theta}_+\Gamma^\mu\Gamma^{\nu lmn}\Gamma^\sigma\theta_-) + 3(\partial_j\overline{\theta}_-\Gamma_{11}\theta_+)(\partial_k\overline{\theta}_+\Gamma^{\nu lmn}\theta_-) + (\partial_k\overline{\theta}_+\Gamma_{11}\Gamma^r\theta_+) \\
&(\partial_j\overline{\theta}_-\Gamma_r\Gamma^{\nu lmn}\theta_-) + 3(\overline{\theta}_+\Gamma_{r\sigma}\partial_j\theta_-)(\partial_k\overline{\theta}_+\Gamma^r\Gamma^{\nu lmn}\Gamma_{11}\Gamma^\sigma\theta_-) + (\overline{\theta}_+\Gamma^\mu\partial_k\theta_+) \\
&(6(\partial_j\overline{\theta}_-\Gamma_\mu\Gamma^{\nu lmn}\Gamma_{11}\theta_-) + 5(\partial_j\overline{\theta}_-\Gamma^{\nu lmn}\Gamma_{11}\Gamma_\mu\theta_-)) + 2(\overline{\theta}_-\Gamma_{rs}\partial_k\theta_-) \\
&(\overline{\theta}_+\Gamma^r\Gamma^{\nu lmn}\Gamma_{11}\Gamma^s\partial_j\theta_+)] \qquad (5.11)
\end{aligned}
$$

$$
\begin{aligned}
\overline{F}_{fglmn} = \frac{1}{96\times 5!} T_{NR}\epsilon_{0jk}[3(\overline{\theta}_-\Gamma^\mu\partial_j\theta_-)(-5(\overline{\theta}_-\Gamma_{\mu fglmn}\partial_k\theta_+) + \\
4(\overline{\theta}_+\Gamma_{\mu fglmn}\partial_k\theta_-)) + (\overline{\theta}_-\Gamma^a\partial_j\theta_+)(17(\overline{\theta}_-\Gamma_{fglmn}\Gamma_a\partial_k\theta_-) - 22(\overline{\theta}_-\Gamma_a\Gamma_{fglmn}\partial_k\theta_-)) + \\
4(\overline{\theta}_+\Gamma^a\partial_j\theta_-)(\overline{\theta}_-\Gamma_{fglmn}\Gamma_a\partial_k\theta_- - 5(\overline{\theta}_-\Gamma_a\Gamma_{fglmn}\partial_k\theta_-)) + 9(\overline{\theta}_-\Gamma_{11}\Gamma^\mu\partial_k\theta_+) \\
(\partial_j\overline{\theta}_-\Gamma_{11}\Gamma_{\mu fglmn}\theta_-) + 12(\overline{\theta}_+\Gamma_{11}\Gamma^\mu\partial_k\theta_-)(\partial_j\overline{\theta}_-\Gamma_{\mu fglmn}\Gamma_{11}\theta_-) + (\partial_j\overline{\theta}_-\Gamma_{11}\Gamma^a\theta_-) \\
(-4(\partial_k\overline{\theta}_+\Gamma_{11}\Gamma_{fglmn}\Gamma_a\theta_-) + 13(\partial_k\overline{\theta}_+\Gamma_{11}\Gamma_a\Gamma_{fglmn}\theta_-) + 6(\partial_k\overline{\theta}_-\Gamma_{11}\Gamma_a\Gamma_{fglmn}\theta_+) + \\
2(\partial_k\overline{\theta}_-\Gamma_{11}\Gamma_{fglmn}\Gamma_a\theta_+)) - (\partial_k\overline{\theta}_-\Gamma_{11}\Gamma^a\Gamma_{fglmn}\Gamma^b\theta_-)(4(\partial_j\overline{\theta}_-\Gamma_b\Gamma_a\Gamma_{11}\theta_+) + \\
3(\partial_j\overline{\theta}_+\Gamma_b\Gamma_a\Gamma_{11}\theta_-)) + 3(\overline{\theta}_-\Gamma_b\Gamma_a\partial_j\theta_-)(\partial_k\overline{\theta}_+\Gamma^b\Gamma_{fglmn}\Gamma^a\theta_-) + 2(\overline{\theta}_-\Gamma^{ab}\partial_k\theta_-) \\
(\overline{\theta}_-\Gamma_a\Gamma_{fglmn}\Gamma_b\partial_j\theta_+ + \overline{\theta}_+\Gamma_a\Gamma_{fglmn}\Gamma_b\partial_j\theta_-) - 6(\overline{\theta}_-\Gamma_{11}\partial_k\theta_+)(\partial_j\overline{\theta}_-\Gamma_{11}\Gamma_{fglmn}\theta_-) + \\
12(\partial_j\overline{\theta}_-\Gamma_{11}\theta_+)(\partial_k\overline{\theta}_-\Gamma_{11}\Gamma_{fglmn}\theta_-) - 3(\partial_k\overline{\theta}_+\Gamma_{11}\Gamma_{fglmn}\Gamma^{\mu a}\theta_-)(\overline{\theta}_-\Gamma_{11}\Gamma_{\mu a}\partial_j\theta_-) - \\
(\overline{\theta}_-\Gamma^{\mu a}\partial_k\theta_+)(4(\partial_j\overline{\theta}_-\Gamma_{\mu a}\Gamma_{fglmn}\theta_-) - \partial_j\overline{\theta}_-\Gamma_{fglmn}\Gamma_{\mu a}\theta_-) - 2(\overline{\theta}_+\Gamma^{\mu a}\partial_k\theta_-) \\
(\overline{\theta}_-\Gamma_{\mu a}\Gamma_{fglmn}\partial_j\theta_- + \partial_j\Gamma_{\mu a}\Gamma_{fglmn}\theta_-)] - \\
\frac{1}{192\times 5!}\epsilon_{0jk}\epsilon_{defglmn}\delta^{ba}_{de}[(16(\overline{\theta}_+\Gamma_c\partial_j\theta_-) + 21(\overline{\theta}_-\Gamma_c\partial_j\theta_+))(\overline{\theta}_-\Gamma^{abc}\Gamma_{11}\partial_k\theta_-) + \\
8(7(\overline{\theta}_+\Gamma^a\partial_j\theta_-) + 3(\overline{\theta}_-\Gamma^a\partial_j\theta_+))(\overline{\theta}_-\Gamma^b\Gamma_{11}\partial_k\theta_-) - (\overline{\theta}_-\Gamma_\mu\partial_j\theta_-) \\
(12(\overline{\theta}_+\Gamma^{\mu ab}\Gamma_{11}\partial_k\theta_-) + 11(\overline{\theta}_-\Gamma_{11}\Gamma^{\mu ab}\partial_k\theta_+)) + 8(\overline{\theta}_-\Gamma_\mu\Gamma^b\Gamma_{11}\partial_j\theta_-)(\overline{\theta}_+\Gamma^{\mu a}\partial_k\theta_- + \\
\overline{\theta}_-\Gamma^{\mu a}\partial_k\theta_+) + (\partial_k\overline{\theta}_+\Gamma_{\mu c}\theta_-)(\partial_j\overline{\theta}_-\Gamma^{\mu abc}\Gamma_{11}\theta_- + 2(\overline{\theta}_-\Gamma^{\mu b}\Gamma_{11}\partial_j\theta_-)\eta^{ac}) + \\
(\overline{\theta}_-\Gamma_{11}\Gamma_c\partial_k\theta_-)(-8(\overline{\theta}_+\Gamma^{abc}\partial_j\theta_-) + 3(\overline{\theta}_-\Gamma^{abc}\partial_j\theta_+)) + (\overline{\theta}_-\Gamma^{\mu ab}\partial_j\theta_-) \\
(12(\overline{\theta}_+\Gamma_{11}\Gamma_\mu\partial_k\theta_-) - 17(\overline{\theta}_-\Gamma_{11}\Gamma_\mu\partial_k\theta_+)) - 4(\partial_j\overline{\theta}_-\Gamma_{cd}\Gamma_{11}\theta_+)(\partial_k\overline{\theta}_-\Gamma^c\Gamma^{ab}\Gamma^d\theta_-) + \\
(\partial_j\overline{\theta}_+\Gamma_{cd}\Gamma_{11}\theta_-)(2(\partial_k\overline{\theta}_-\Gamma^{cd}\Gamma^{ab}\theta_-) - 17(\overline{\theta}_-\partial_k\theta_-)\eta^{ac}\eta^{bd}) + (\overline{\theta}_-\Gamma_{cd}\partial_k\theta_-) \\
(11(\partial_j\overline{\theta}_+\Gamma_{11}\theta_-)\eta^{ac}\eta^{bd} + 2(-\partial_j\overline{\theta}_+\Gamma^{cd}\Gamma^{ab}\Gamma_{11}\theta_- + \overline{\theta}_-\Gamma^c\Gamma^{ab}\Gamma^d\Gamma_{11}\partial_j\theta_+ - \\
\overline{\theta}_+\Gamma^c\Gamma^{ab}\Gamma^d\Gamma_{11}\partial_j\theta_-)) + 3(\overline{\theta}_-\Gamma_{\mu c}\Gamma_{11}\partial_j\theta_-)(\partial_k\overline{\theta}_+\Gamma^{\mu abc}\theta_- + 2(\partial_k\overline{\theta}_+\Gamma^{\mu b}\theta_-)\eta^{ac})]
\end{aligned}
\tag{5.12}
$$

References

[1] N. Berkovits, Super-Poincaré covariant quantization of the superstring, JHEP **04** (2000) 018 (hep-th/0001035).

[2] N. Berkovits, Cohomology in the pure spinor formalism for the superstring, JHEP **09** (2000) 046 (hep-th/0006003).

[3] N. Berkovits, Covariant quantization of the superparticle using pure spinors, JHEP **09** (2001) 016 (hep-th/0105050).

[4] N. Berkovits, ICTP lectures on covariant quantization of the superstring (hep-th/0209059).

[5] O. Chandía and B.C. Vallilo, Conformal invariance of the pure spinor superstring in a curved background, JHEP **0404** (2004) 041 (hep-th/0401226).

[6] O. Chandía, A note on the classical BRST symmetry of the pure spinor string in a curved background, JHEP **0607** (2006) 019 (hep-th/0604115).

[7] O.A. Bedoya and O. Chandía, One loop conformal invariance of the type II pure spinor superstring in a curved background, JHEP **0701** (2007) 042 (hep-th/0609161).

[8] J. Kluson, Note about redefinition of BRST operator for pure spinor strings in general background, Phys. Rev. D **78** (2008) 066011 (arXiv:0803.4390 [hep-th]).

[9] C. Mafra, Superstring scattering amplitudes with the pure spinor formalism", PhD Thesis (arXiv:0902.1552 [hep-th]).

[10] C. Mafra and C. Stahn, The one-loop open superstring massless five-point amplitude with the non-minimal pure spinor formalism, JHEP **0903** (2009) 126 (arXiv:0902.1539 [hep-th]).

[11] N. Berkovits and C. Mafra, Equivalence of the two-loop superstring amplitudes in the pure spinor and RNS formalisms, Phys. Rev. Lett. **96** (2006) 011602 hep-th/0509234.

[12] N. Berkovits and C. Mafra, Some superstring amplitude computations with the non-minimal pure spinor formalism, JHEP **0611** (2006) 079 (hep-th/0607187).

[13] C. Mafra and O. Schlotterer, Cohomology foundations of one-loop amplitudes in pure spinor superspace, (arXiv:1408.3605 [hep-th]).

[14] P. Fré and P. A. Grassi, Pure Spinors, Free Differential Algebras, and the Supermembrane, Nucl. Phys. B **763** (2007) 1 (hep-th/0606171).

[15] J. Gomis and H. Ooguri, Nonrelativistic closed string theory, J. Math. Phys. **42** (2001) 3127 (hep-th/0009181).

[16] J. Brugues, T. Curtright, J. Gomis, L. Mezincescu, Non-relativistic strings and branes as non-linear realizations of Galilei groups, Phys. Lett. B **594** (2004) 227 (hep-th/0404175).

[17] J. Gomis, F. Passerini, T. Ramirez and A. van Proeyen, Non relativistic Dp branes, JHEP **0510** (2005) 007 (hep-th/0507135).

[18] K. Kamimura , T. Ramirez, Brane dualities in non-relativistic limit, JHEP **0603** (2006) 058 (hep-th/0512146).

[19] M. Aganagic, C. Popescu, J.H. Schwarz, D-brane actions with local kappa symmetry, Phys. Lett. B **353** (1997) 311 (hep-th/9610249).

[20] M. Aganagic, C. Popescu, J.H. Schwarz, Gauge invariant and gauge fixed D-brane actions, Nucl. Phys. B **495** (1997) 99 (hep-th/9612080).

[21] M. Hatsuda, K. Kamimura, Wess-Zumino actions for IIA D-branes and their supersymmetries, Nucl. Phys. B **535** (1998) 499 (hep-th/9804087).

[22] P.A. Grassi, G. Policastro and P. van Nieuwenhuisen, An introduction to the covariant quantization of the superstring, Class. Quant. Grav. **20** (2003) S395 (hep-th/0302147).

[23] P.A. Grassi, G. Policastro and P. van Nieuwenhuisen, Yang-Mills theory as an illustration of the covariant quantization of superstrings, in *3rd International Sakharov Conference on Physics,*, 24-29 June, 2002, Moscow, Russia (hep-th/0211095).

[24] J. Hoogeveen, *Fundamentals of the Pure Spinor Formalism*, Amsterdam University Press, Amsterdam, 2010.

[25] L. Anguelova, P.A. Grassi, Super D-branes from BRST symmetry, JHEP **0311** (2003) 010 (hep-th/0307260).

[26] L. Brink and M. Henneaux, *Principles of String Theory*, Plenum Press, New York and London, 1988.